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ITERATIVE ESTIMATION OF NORMAL POPULATION MEAN USING COMPUTATIONAL-STATISTICAL INTELLIGENCE

Ashok Sahai, Raghunadh M. Acharya**
Department of mathematics and statistics; Faculty of science and technology. St. Augustine Campus; the University of the West Indies. Trinidad & Tobago; West Indies

Abstract. This paper addresses the issue of finding the most efficient estimator of the normal population mean when the population "Coefficient of Variation (C. V.)" is 'Rather-Very-Large' though unknown, using a small sample (sample-size ≤ 30). The paper proposes an "Efficient Iterative Estimation Algorithm exploiting sample "C. V." for an efficient Normal Mean estimation". The MSEs of the estimators per this strategy have very intricate algebraic expression depending on the unknown values of population parameters, and hence are not amenable to an analytical study determining the extent of gain in their relative efficiencies with respect to the Usual Unbiased Estimator $\bar{X}$(sample mean ~ Say 'UUE'). Nevertheless, we examine these relative efficiencies of our estimators with respect to the Usual Unbiased Estimator, by means of an illustrative simulation empirical study. *MATLAB 7.7.0.471 (R2008b)* is used in programming this illustrative 'Simulated Empirical Numerical Study'.

Key words: MMSE, sample coefficient of variation, complete sufficient statistic, and computational statistics and simulation studies.

1. Introduction.

This paper addresses the issue of finding an optimal estimator of the normal population mean when the population 'Coefficient of Variation (C.V.)', say "a" is unknown and is expected to be 'Rather-Very-Large' (Say, because population standard deviation $\sigma \geq 5$), as per the pilot surveys of the population at hand. The cases of such a high population "standard deviation σ" are not very uncommon, for example the study-variable is the weights of fishes in an ocean. Beside this, as is well-known, the 'C.V.' is invariant under the change of scale of the study variable, say X; but is not so in case of the variable's translation. Therefore, for example, if we have a good estimate of the population mean and translate the study variable by adding say, '0.1' after subtracting this estimate, and hence, consequently (to a great extent, without loss of any generality), we could presume that the 'MEAN' of our parent population under study say, "θ" is say, "0.1". The new 'population "C.V." of this translated parent population will be more-or-less about 'ten-fold' of that of the original translated population, if the mean was 1. Further, in this paper we have considered the case of small sample-size ≤ 30.

Devore, Jay L. (1982) detailed the optimal estimation procedures in statistical estimation. Searls (1964), Khan (1968), Gleser and Healy (1976), Arnholt and Hebert (1995), and Ashok Sahai (2011) considered the problem of estimating the normal population mean and variance, when its Coefficient of Variation (C. V.) "a" is known.

It is very well- known that the Searles (1964)'s Minimum Mean Squared Error (MMSE) Estimator for the Normal Population N (θ, σ^2)'s mean 'θ' gets to be: Say, SE = $\overline{X}$ / $(1+a^2/n)$; Wherein, 'a' is the Known Population Coefficient of Variation (σ/θ) & 'n' is the size of the random sample from the Normal Population with the sample mean " $\overline{X} = (\sum_{i=1}^{i=n} X_i)/n$ ".

The Searles (1964)'s estimator is known to be "MMSE", and hence optimal. But this estimator could be used only when for such a Normal population, $N(\theta, a\theta^2)$, 'a' is known. The Usual Unbiased Estimator (UUE) of population mean "θ" is well known to be the sample mean & variance: -" $\overline{X} = (\sum_{i=1}^{i=n} X_i)/n$ ". ≡ UUE of θ & $S^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2/(n-1)$ ≡ UUE of $a\theta^2$.

In this case of known 'CV', the sufficient statistic ($\bar{X}$; S^2) is not complete. Consequently, when estimating mean/variance, or essentially any function of unknown parameter θ, we are groping in the dark when attempting to find its Universally Minimum Mean Squared Error Estimate (UMMSE), or to find the Universally Minimum Variance Unbiased Estimate (UMVUE).

However, in practice, more often than not, the population 'C.V.' "a" is rather unknown. Winston A. Richards, Robin Antoine, Ashok Sahai, and M. Raghunadh Acharya (2010), Richards, Winston A., Ashok Sahai, Robin Antoine, Kimberly Wright, and Raghunadh M. Acharya (2009) & Miodrag M Lovric & Ashok Sahai (2011) considered using the sample coefficient of variation for efficient estimation of Normal population parameter θ, and hence of the mean and variance.

As noted earlier, it is very well-known that the Searles (1964) Estimator for 'θ' gets to be:

Say, SE = $(\overline{X})$/ $(1+V^2)$; (1.1)

Wherein 'V' is square of the coefficient of variation of '$(\overline{X})$'. (1.2)

As such, [C. V.$(\overline{X})$]^2 = $\sigma^2/(\sqrt{n}.\ \theta)^2 \equiv V = a/n$. (1.3)

In this paper we have, however, been motivated by this result of Searls (1964) [9] leading to the estimator "SE", as in (1.1) for the estimation of the normal population mean when 'C.V.' is unknown.

As noted earlier, the fact that 'V' might not be known is a reality much more often in practice than when it is known.

In fact the assumption that 'V' is quite closely known is also seldom justified. In the absence of the knowledge of 'a', we propose to use its sample counterpart:

Say, $v = \hat{\mathbf{a}} = S^2/[n.\bar{X}^2]$; $S^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2/(n-1) \equiv$ UUEV (1.4)

In the above, we use the Usual Unbiased Estimate of the population Variance σ^2 (UUEV), namely the sample variance S^2.

2. The Proposed Estimators.

Before we propose our "Iterative Algorithm for Efficient Estimation of the Mean of a Normal Population" using 'Computational-Statistical Intelligence' & sample counterpart 'v' of the unknown C.V. "a", we would recall that the Usual Unbiased Estimator of the normal population mean "θ" of N (θ, $a\theta^2$) is [which is consistent, unbiased efficient & sufficient estimator for "θ"]:

"$\overline{X} = (\sum_{i=1}^{i=n} X_i)/n$" ≡ UUE. (2.1)

Now, we propose estimator say, Mean-Estimator Sahai-Acharya (MEANESA (0)), the beginner [Sample Counterpart of the Searles (1964)'s estimator} of the "Iterative Algorithm":

MEANESA (0) = $\bar{X}$ / (1+v); wherein v = S^2/ {n. ($\bar{X}$)^2} (2.2)

Inasmuch as the simulation study brings forth the fact that "MEANESA (0) ≡ $\bar{X}$ / (1+v)" is significantly more efficient than UUE '$\bar{X}$' [for small sample-sizes]; we are encouraged to try the following series of estimators obtained by using $\bar{X}$ / (1+v) in place of $\bar{X}$, successively.

MEANESA [1] = $\bar{X} / (1+v)^2$ (2.3)

MEANESA [2] = $\bar{X} / (1+v)^3$ (2.4)

MEANESA [3] = $\bar{X} / (1+v)^4$ (2.5)

MEANESA [4] = $\bar{X} / (1+v)^5$ (2.6)

MEANESA [5] = $\bar{X} / (1+v)^6$ (2.7)

MEANESA [6] = $\bar{X} / (1+v)^7$ (2.8)

MEANESA [7] = $\bar{X} / (1+v)^8$ (2.9)

MEANESA [8] = $\bar{X} / (1+v)^9$ (2.10)

MEANESA [9] = $\bar{X} / (1+v)^{10}$ (2.11)

And our "Iterative Efficient Mean-Estimator of Sahai-Acharya at Iteration # I": Say, MEANESA (I) [I = 1 (1) m (Positive Integer)]. These are generated with the recurrence relation: ~

MEANESA (I) = (1 + k)* MEANESA [I-1] – k* UUEV; (2.12)

Wherein "k" is 'Design-Parameter' of the proposed "Iterative Efficient Mean-Estimator Procedure of Sahai-Acharya".

To determine the "Optimal Value" of "k", the 'Design-Parameter' of the proposed "Iterative Efficient Mean-Estimator Procedure of Sahai-Acharya" an extensive simulation-study was carried out. Using the "Computational Intelligence" gained thorough the aforesaid extensive simulation-study, it was discovered that the best choice/optimal choice for "k", the 'Design-Parameter', happens to be "0.1". Subsequently, to determine the "Optimal Value" for "I" [The Number of Iteration to STOP at]; again we use "Computational Statistics/Simulation Study", as is illustrated in the following section of "Simulation Study".

We start by noting that the aforesaid estimator "MEANESA (I)" involves very intricate algebraic expression depending on the unknown values of 'θ' & 'a'!

Also, the proposed estimators "MEANESA (I)" have no close-form expression for their MSE [Mean Squared-Error], and thus are not amenable to any analytical study determining their relative efficiency as compared to the UUE/ Usual Unbiased sample mean Estimator " $\overline{X}$ " of the population mean "θ".

Therefore, the only open recourse is to study this problem numerically with a large number of simulated samples of various illustrative sizes (Say 'n') from the parent normal population with illustrative values of the population mean and standard deviation. This type of "Simulated Numerical Empirical Study" has been attempted later in the subsequent section considering a large number *"55,555" of Replications,* via simulated samples.

Now, to discover improvement brought forth by these iteratively generated estimators proposed in our paper, we have taken to an empirical simulation study in the following section.

We define the Relative Efficiency, REFF (•) of the estimator "•" at hand, relative to $\overline{X}$ ≡ UUE in equation (2.1) as follows.

$$Reff(\bullet) = [100.MSE(\bullet) \,/\, V(\overline{X})]\ \%;\ \text{wherein MSE}(\bullet) = E[\bullet - \theta]^2. \qquad (2.13)$$

In above, $MSE(\bullet) = V(\bullet) + B^2(\bullet)$ stands for the MSE {Mean Square Error} of the estimator at hand, $V(\bullet)$ for its variance, and $B(\bullet)$ for its bias.

In this context, we mention that we have considered the estimators for their 'Relative Efficiencies' as per the "Iteration #", # = 0 (1) 9, for the illustration.

3. The Simulation/Empirical Numerical Study

In the preceding section, it is apparent, from the fact that in the absence of any 'closed form analytical expressions' of the *Reff* (•)'s facilitating any feasible comparisons, that the answer to the question as to what is the extent of the relative gain/ achievement in pursuing the 'Iteratively More Efficient Estimation' of the normal population "MEAN", lies in trying to know it through an illustrative *"Simulation/Empirical Numerical Study"*, as is attempted in this section. These *Reff* (•)'s have been calculated for *EIGHT* illustrative values of the σ ≡ *5.0, 5.5, 6.0, 6.5, 7.0, 8.0, 9.0 &10.0.* The "Population Mean" is envisaged in our paper to be very small: θ ~0.10. As also, we have carried out the illustrative numerical study for *SIX* example-values of the sample size, namely *n =5, 10, 15, 20, 25, and 30.* The values of the actual MSE's are calculated by considering the random samples of size 'n' using *'55,555' Replications* (pseudo-random normal samples of size 'n') for various estimators "MEANESA (I)" with I = 0 (1) 9, as also for the Usual Unbiased Estimator (UUE), namely the sample mean " $\overline{X}$ ". Hence the values of Reff (•)'s are calculated as per (2.13). These Reff (•)'s are reported, to the closest first decimal place of their respective actual values, in the *SIX* tables in the *APPENDIX,* corresponding to each of the *SIX* illustrative values of the sample-size 'n'. *MATLAB 7.7.0 (R2008b)* is used in programming the calculations in this illustrative *'Simulated Empirical Numerical Study'.*

4. Conclusions.

As expected, the "Relative Efficiencies" of the proposed "Iteratively More Efficient" estimators of the 'Normal Population Mean' are progressively better with the increase of the "Iteration Number (#) I" in the impugned estimator MEANESA (I), till a certain stage depending on the values of the sample size n and the population parameter σ. In practice, we could use the values of our sample mean and that of the sample variance, namely that of " $\bar{X}$ and S^2", in place of "θ & σ^2", and using these values and the relevant sample size value 'n', we could conduct the "Simulation Study" outlined in the preceding section. This "Simulation Study" could be used to determine the "Optimal Number of the Iterations I", where we must stop to have the "Most Efficient Estimator" of the population mean $\overline{X}$, using this "Computational Intelligence" available per the 'Simulation Study". Incidentally, we note that the smaller the "n", the greater the "Optimal Number of the Iterations I".

Though we have limited the illustration up to I = 9, clearly the indications are well-supportive of the fact that we could do better by proceeding further to generate progressively more efficient estimators, till we reach the "Optimal Number of the Iterations I" where we must stop to have the 'Most Efficient Estimator' of the population variance σ^2, as to be determined by using this "Simulation Study"!

This achievement, as is illustrated through the results in the Tables in the APPENDIX, has been the motivating aim behind this paper.

It is very note-worthy to observe the results tabulated in Tables A.1 to Table A.3 Vis-à-Vis those tabulated in Tables A.4 to Table A.6. As the sample-size gets to be significant (>=20) the asymptotic property of the estimator phases out the gainful effect of "Iteration" unless the standard deviation 'σ' happens to be rather very large!

It might be remarked in this context, that the extent-and-the-stage till which the proposed "Algorithm" of generating iteratively more efficient estimator depends on the value of the coefficient-of-variation "V" [i.e., the C. V. ($\bar{X}$)] as would be available through the value of its sample counterpart "v". Precisely, this value of "v" might be used with the value of " $\bar{X}$ " [as an estimate of θ] together with that of S^2 [as an estimate of σ^2]. To seek the guidance about the "Optimal Value of 'I' for a particular sample-size 'n' we have to have this "Simulation Study" using " $\bar{X}$ & S^2" in place of "θ & σ^2" for generating the pseudo-random samples of size 'n' from N (θ, σ) with a 'Replication of 55,555'…etc.

Thus, "Efficient Iterative Estimation Algorithm, exploiting sample coefficient of variation, for the Efficient Normal Variance Estimation" is essentially in the arena of 'Computational Statistics', an upcoming area of "Statistics" exploiting the 'Electronic Computers' like in other areas, e.g. 'Mathematics', 'Physics', 'Chemistry'; inseminating 'Computational Mathematics', 'Computational Physics', 'Computational Chemistry', etc.

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ITERATYVUS NORMALINIO POPULIACIJOS VIDURKIO ĮVERTINIMAS TAIKANT SKAIČIUOJAMOSIOS STATISTIKOS METODUS

Ashok Sahai, Raghunadh M. Acharya

Santrauka

Straipsnyje analizuojama efektyvaus normalinio populiacijos vidurkio įvertinio radimo problema, kai populiacijos variacijos koeficientas yra "labai didelis" arba nežinomas. Tyrimui naudojama maža imtis (imties elementų skaičius ≤ 30). Straipsnyje pasiūlytas iteracinis įvertinio radimo algoritmas, kuris naudoja imties variacijos koeficientą efektyviam normalinio vidurkio įvertinimui.

Šioje strategijoje įvertinių mažiausia kvadratinė paklaida (MSE) turi sudėtingą algebrinę išraišką, priklausomą nuo nežinomų populiacijos parametrų verčių ir todėl būtų sudėtingą ją įvertinti analitiniu būdu nustatant jų santykinio efektyvumo įprastinio įvertinio, neturinčio poslinkio, $\bar{X}$ atžvilgiu, naudos mastą. Straipsnyje pristatytas santykinių įvertinių efektyvumo įprastinių neturinčių poslinkio įvertinių atžvilgiu tyrimas, panaudojant iliustracinę empirinių duomenų simuliaciją. Tyrimui atlikti buvo pasitelkta *MATLAB 7.7.0.471 (R2008b)* programinė įranga.

Pagrindiniai žodžiai: MMSE, imties variacijos koeficientas, statistika, modeliavimas.

APPENDIX.

Table a.1. [n = 5].

*relative efficiencies of normal mean estimator (•) [in %]: reffs (•) for various σ's with θ = 0.1↓

estrs↓\σ →	5.0	5.5	6.0	6.5	7.0	8.0	9.0	10.0
meanesa(0)	660.2	679.8	677.4	673.3	664.6	682.8	2551.8	676.8
meanesa (1)	2231.3	2403.0	2405.6	2423.5	2365.6	2572.4	6526.0	2557.8
meanesa (2)	4674.8	5273.4	5414.6	5699.7	5586.8	6511.7	12528.2	6754.9
meanesa (3)	6952.4	8107.7	8671.7	9634.4	9689.9	12003.2	18859.8	13607.3
meanesa (4)	8380.9	9943.3	11040.5	12746.7	13297.1	17082.7	23772.5	21534.2
meanesa (5)	9102.9	10882.3	12400.0	14588.0	15725.4	20505.2	26870.9	28198.4
meanesa (6)	9436.9	11319.4	13105.8	15522.6	17135.7	22426.6	28618.5	32624.6
meanesa (7)	9586.7	11517.7	13463.0	15965.8	17904.8	23423.2	29556.5	35180.1
meanesa (8)	9652.6	11607.3	13644.8	16169.3	18316.6	23928.2	30051.0	36554.5
meanesa (9)	9680.7	11647.6	13738.3	16259.8	18537.4	24183.8	30721.0	37269.0

*relative to the usual unbiased estimator of normal mean ~ uue.

Table a.2. [n = 10].

*relative efficiencies of normal mean estimator (•) [in %]: reffs (•) for various σ's with θ = 0.1↓

estrs↓\σ →	5.0	5.5	6.0	6.5	7.0	8.0	9.0	10.0
meanesa(0)	1135.8	1184.5	1249.2	1295.2	1324.8	1392.3	1419.1	1499.3
meanesa (1)	2296.2	2663.3	3089.2	3454.9	3814.5	4622.8	5304.1	6239.1
meanesa (2)	2479.5	2967.9	3534.3	4065.9	4656.9	6007.5	7435.6	9111.7
meanesa (3)	2481.9	2988.3	3573.4	4136.7	4784.8	6252.3	7888.0	9697.4
meanesa (4)	2471.9	2979.7	3564.9	4132.8	4793.4	6279.1	7950.6	9763.5
meanesa (5)	2465.5	2972.7	3556.6	4124.3	4787.6	6274.8	7948.0	9753.6
meanesa (6)	2462.1	2968.7	3551.7	4118.7	4782.5	6269.0	7939.0	9740.0
meanesa (7)	2460.4	2966.6	3549.2	4115.5	4779.3	6265.0	7932.3	9731.2
meanesa (8)	2459.5	2965.4	3548.0	4113.7	4777.5	6262.6	7928.2	9726.2
meanesa (9)	2459.0	2964.8	3547.3	4112.8	4776.5	6261.2	7925.8	9723.5

*relative to the usual unbiased estimator of normal mean ~ uue.

Table a.3. [n = 15].

*relative efficiencies of normal mean estimator (•) [in %]: reffs (•) for various σ's with θ = 0.1↓

estrs↓\σ →	5.0	5.5	6.0	6.5	7.0	8.0	9.0	10.0
meanesa(0)	976.3	1093.3	1207.0	1332.9	1448.9	1640.5	1792.1	1950.4
meanesa (1)	1130.8	1358.5	1593.9	1868.8	2165.0	2753.1	3419.2	4159.4
meanesa (2)	1104.8	1338.2	1580.4	1858.2	2163.9	2793.1	3530.4	4345.9
meanesa (3)	1093.2	1325.5	1567.4	1841.8	2145.4	2776.3	3515.4	4327.2
meanesa (4)	1089.0	1320.7	1562.2	1835.1	2137.5	2768.0	3505.4	4312.7
meanesa (5)	1087.6	1318.9	1560.2	1832.6	2134.5	2764.7	3501.2	4306.4
meanesa (6)	1087.0	1318.2	1559.4	1831.6	2133.3	2763.5	3499.4	4303.8
meanesa (7)	1086.8	1317.9	1559.1	1831.3	2132.9	2763.0	3498.7	4302.8
meanesa (8)	1086.7	1317.8	1558.9	1831.1	2132.7	2762.7	3498.3	4302.3
meanesa (9)	1086.7	1317.8	1558.9	1831.0	2132.6	2762.6	3498.2	4302.2

*relative to the usual unbiased estimator of normal mean ~ uue.

Table a.4. [n = 20].

*relative efficiencies of normal mean estimator (•) [in %]: reffs (•) for various σ's with θ = 0.1↓

estrs↓\σ →	5.0	5.5	6.0	6.5	7.0	8.0	9.0	10.0
meanesa(0)	666.7	788.1	894.9	1026.7	1147.7	1385.2	1623.3	1826.7
meanesa (1)	639.9	774.4	907.7	1079.0	1236.8	1609.2	2048.7	2470.7
meanesa (2)	622.3	753.5	885.7	1054.2	1208.6	1579.3	2018.4	2449.5
meanesa (3)	617.5	747.8	879.5	1046.8	1200.0	1568.7	2004.8	2435.3
meanesa (4)	616.2	746.2	877.7	1044.8	1197.6	1565.6	2000.7	2430.9
meanesa (5)	615.7	745.7	877.2	1044.2	1196.9	1564.7	1999.5	2429.5
meanesa (6)	615.6	745.6	877.0	1044.0	1196.7	1564.4	1999.1	2429.1
meanesa (7)	615.5	745.5	877.0	1044.0	1196.7	1564.3	1998.9	2429.0
meanesa (8)	615.5	745.5	876.9	1043.9	1196.6	1564.3	1998.9	2428.9
meanesa (9)	615.5	745.5	876.9	1043.9	1196.6	1564.2	1998.9	2428.9

*relative to the usual unbiased estimator of normal mean ~ uue.

Table a.5. [n = 25].

*relative efficiencies of normal mean estimator (•) [in %]: reffs (•) for various σ's with θ = 0.1↓

estrs↓\σ →	5.0	5.5	6.0	6.5	7.0	8.0	9.0	10.0
meanesa(0)	448.7	531.1	631.6	730.4	826.8	1028.3	1257.3	1493.1
meanesa (1)	402.9	481.6	584.0	689.5	790.1	1022.2	1302.8	1611.1
meanesa (2)	393.3	470.3	570.8	674.1	772.7	1001.7	1277.4	1582.5
meanesa (3)	391.2	468.0	567.9	670.6	768.8	997.0	1271.2	1575.3
meanesa (4)	390.7	467.4	567.3	669.8	767.9	995.8	1269.6	1573.6
meanesa (5)	390.6	467.3	567.1	669.6	767.6	995.6	1269.2	1573.1
meanesa (6)	390.6	467.2	567.1	669.5	767.6	995.5	1269.1	1573.0
meanesa (7)	390.6	467.2	567.0	669.5	767.5	995.5	1269.1	1573.0
meanesa (8)	390.5	467.2	567.0	669.5	767.5	995.5	1269.1	1573.0
meanesa (9)	390.5	467.2	567.0	669.5	767.5	995.4	1269.1	1573.0

*relative to the usual unbiased estimator of normal mean ~ uue.

Table a.6. [n = 30].

*relative efficiencies of normal mean estimator (•) [in %]: reffs (•) for various σ's with θ = 0.1↓

estrs↓\σ →	5.0	5.5	6.0	6.5	7.0	8.0	9.0	10.0
meanesa(0)	318.2	379.4	452.9	517.2	600.2	758.6	939.9	1130.6
meanesa (1)	279.7	335.5	406.1	468.3	547.3	705.2	897.9	1107.2
meanesa (2)	274.0	328.8	398.2	459.6	537.0	692.6	882.3	1088.6
meanesa (3)	273.0	327.5	396.8	457.9	535.2	690.3	879.3	1084.9
meanesa (4)	272.7	327.3	396.4	457.6	534.8	689.9	878.7	1084.1
meanesa (5)	272.7	327.2	396.4	457.5	534.7	689.8	878.5	1083.9
meanesa (6)	272.7	327.2	396.3	457.4	534.6	689.7	878.5	1083.9
meanesa (7)	272.7	327.2	396.3	457.4	534.6	689.7	878.5	1083.8
meanesa (8)	272.7	327.2	396.3	457.4	534.6	689.7	878.5	1083.8
meanesa (9)	272.7	327.2	396.3	457.4	534.6	689.7	878.5	1083.8

*relative to the usual unbiased estimator of normal mean ~ uue.