MEASUREMENT SYSTEMS FOR MATHEMATICS AND ASTRONOMY IN ANTIQUITY: PTOLEMY'S CHORD CALCULATION AND NEW CONSIDERATIONS CONCERNING THE "DISCUS IN PLANITIA" ASCRIBED TO ARISTARCHUS. PART I

FLORIN STANESCU

Abstract

The aim of this work is to present new hypotheses concerning a great disk of andesite – approximately 7 m in diameter – which was discovered in Meridional Carpathians in Romania on the sacred terrace of the capital of the ancient kingdom of the Dacians. Having included in this structure all the specific features of a sun dial, the place's latitude φ, the obliquity of the ecliptic in epoch ε, as well as a north–south orientation, is possible and probable that this is the "discus in planitia" that Vitruvius ascribed to Aristarchus.

Key words: sundial, Andesite Sun, Soarele de Andesit, Sarmizegetusa Regia, sacred terrace, Dacian, astronomy in antiquity, discus in planitia.

Introduction

The ancient capital of the Dacian kingdom, Sarmizegetuza Regia, lies in the Orastie Mountains, Romania. Eleven round and rectangular sanctuaries have been discovered so far in this impressive complex of ruins called the Sacred Terrace.

Astronomical knowledge

Several ancient sources report on the astronomical activities of the Dacians. The beginnings of Dacian astronomy are connected to **Zalmoxis** (also known as Zamolxis), a legendary social and religious leader, regarded as a god by the Dacians. According to Herodotus (History, IV, 93-96) Zalmoxis was a slave or disciple of Pythagoras who taught him at Samos. After obtaining his freedom, Zalmoxis returned to his country, where he built a chamber and taught his countrymen of the doctrine of the immortality of the soul. **Strabo's** later report (*Geography* VII, 3,5) adds that Zalmoxis has learnt not only from Pythagoras, but also from the Egyptians, and gives him an ability to "report the will of the gods". Strabo is the first author who maintains that Zalmoxis learnt the science of the skies from Pythagoras:

"*He* [*i.e. Herodotus*] *tells us that one of the Getes named Zamolxis was the slave of Pythagoras and*, during this time, *he had learned from the philosopher the sciences of the sky*. A*fterwards he had assimilated other* knowledge *from the Egyptians, because he also wandered*

about in those places of the world". All later authors stated that Zalmoxis brought back to the Dacians, from Egypt and from Pythagoras, a mystic doctrine regarding the immortality of the soul together with the arts of civilization."

Saint Hippolytus of Rome (in the so-called *Philosophoumena*, II, 2, 22 known also as the *Refutation of All Heresies*) says that Zalmoxis " is said to have taught the Celtic Druids to cultivate the philosophy of Pythagoras" and **Porphyry of Tyre** (in *The Life of Pythagoras*) narrates that "*... Also ...Pythagoras taught him to study the celestial phenomena...".*

But the most conclusive report about Dacian astronomy is to be found in a description by **Iordanes, a 6th-century historian, in** Getica (V, 39, 69-71), of the early history of the Goths (Dacians). His work, in turn, was a summary of the now lost Cassiodorus' much
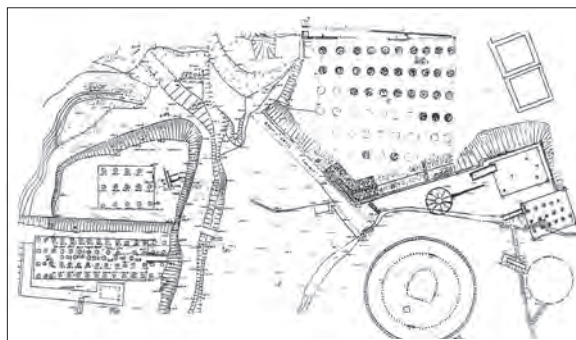

Fig. 1. Map of the zone of the sanctuaries at Sarmizegetusa-Regia, Romania.

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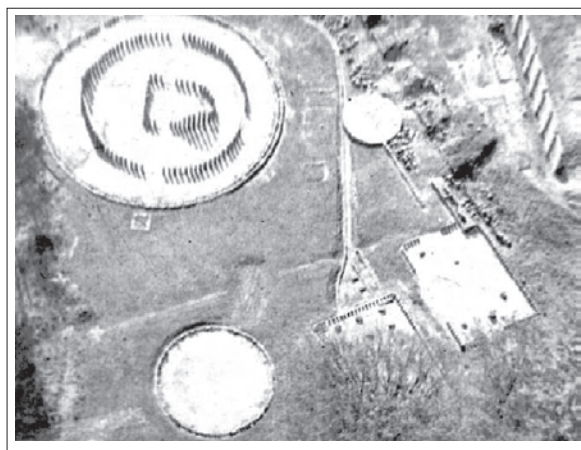ascribed to Aristarchus. Part I

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Fig. 2. The Great Round Sanctuary with the Central Apse, with a diameter of almost 30m and oriented towards sunrise at the winter solstice; the Small Round Sanctuary with a diameter of almost 13m; two other rectangular andesite sanctuaries with north-south orientation; the altar-sundial known as the "The Andesite Sun", and the "stone-ray" indicating the north-south direction.

longer treatment of the history of the Goths (Gaeto-Dacians): "By demonstrating theoretical knowledge he urged them to contemplate the twelve signs and the courses of the planets passing through them, and the whole of astronomy. He told them how the disk of the moon gains increase or suffers loss, and showed them how much the fiery globe of the sun exceeds in size our earthly planet. He explained the names of the three hundred and forty-six stars and told through what signs in the arching vault of the heavens they glide swiftly from their rising to their setting (70). You might have seen one scanning the position of the heavens and another investigating the nature of plants and bushes.

Here stood one who studied the waxing and waning of the moon, while still another regarded the labors of the sun and observed how those bodies which were hastening to go toward the east are whirled around and borne back to the west by the rotation of the heavens".

Ancient Instruments

Numerous compasses made of iron and used for marking circles or measuring small distances have been found at Dacian sites. They are all typical of the epoch (first two centuries AD) and similar to the types known in the Graeco-Roman World (see Fig. 3). It has been observed that while the fastening mechanism for the arms of these compasses is the same as is found generally, its ends allow other arms or tips to be attached, making it different from all other known instruments (cf. Fig. 3A and B; and Fig. 4).

Description of the Research Goals

Following Vitruvius, Ptolemy provided a geometrical means of calculating the sun's position at any date and observer location by the use of constructions called analemmas. An important application of analemmas was the design of **Ptolemy's Coordinate System which** involved the orthogonal projection of the celestial sphere onto three mutually perpendicular planes: the "horizon", the "vertical" and the "meridian" (Fig. 5). Their intersections are called, respectively, the "Meridian line" (M-axis), the "Equinoctial line" (E-axis) and the "Gnomon line" (Z-axis). In order to define spheri-
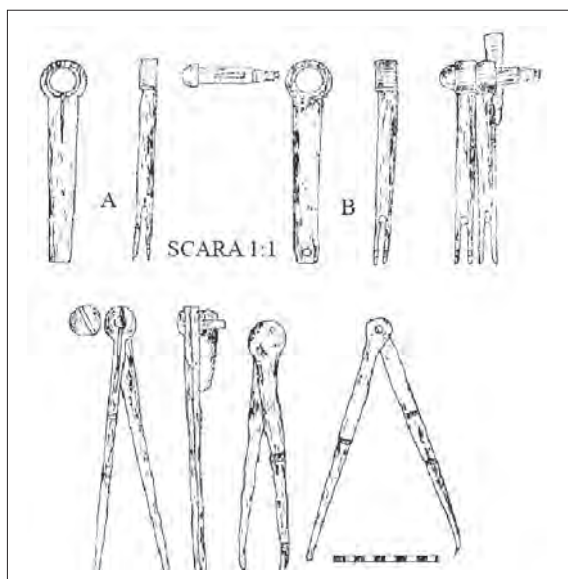

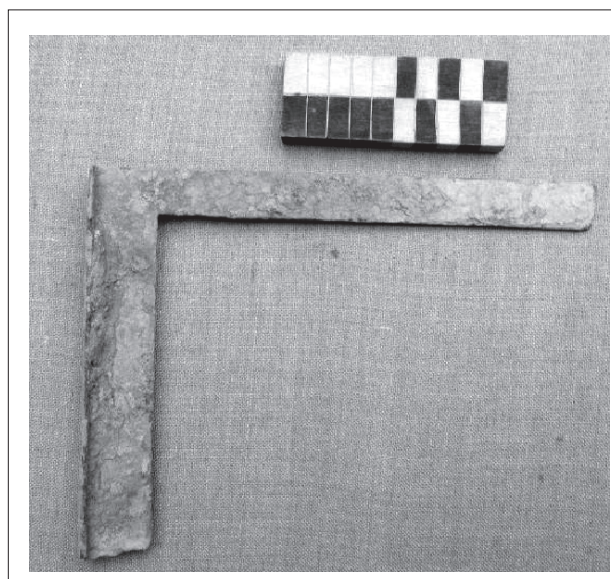Fig.3. Dacian compasses of the "pantographical" type (A and B).
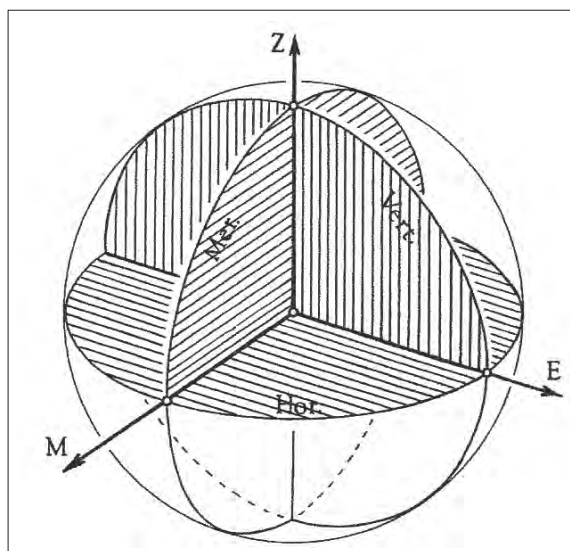
Fig. 4. A Dacian bronze square (triangle).

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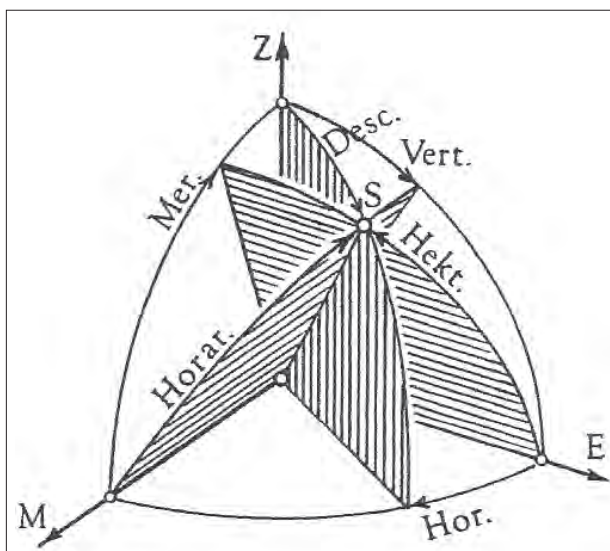Fig. 5. Ptolemy's Coordinate System: three orthogonal planes.

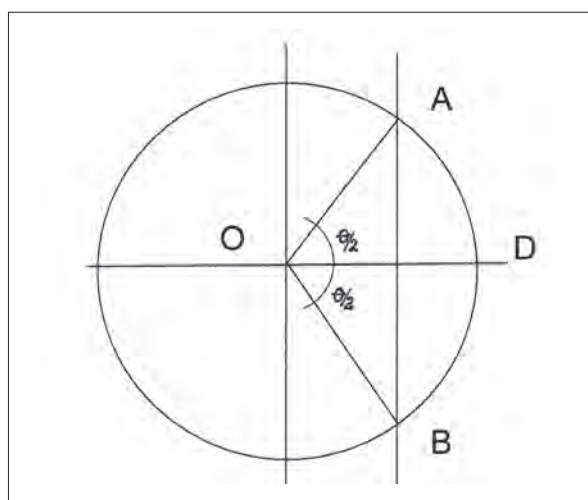Fig. 6. Ptolemy's Coordinate System: three moveable great circles.

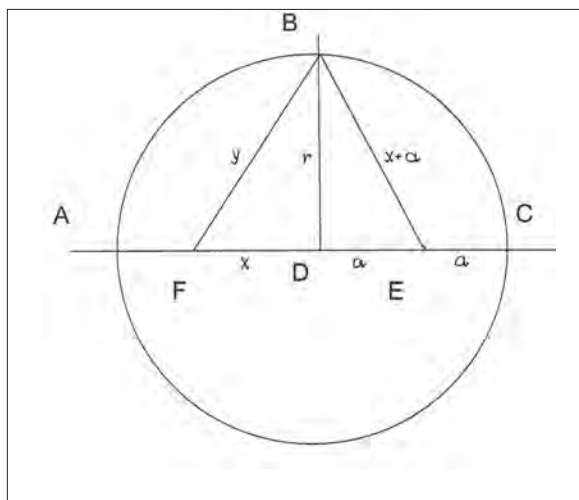Fig. 7. „Crd θ" is the length of the chord AB.

Fig. 8. Chord lengths for the central angles corresponding to the sides of a decagon, hexagon, and pentagon.

cal coordinates for an arbitrary point "S" (this point representing the Sun), Ptolemy (Figure 6) replaced the lines with moveable circles called the hectemoros (the circle that passes through the east and west points and the sun), the horarius (the circle that passes through the meridian circle and the sun) and the descensivus (the circle that passes through the zenith and nadir points and the sun). Analemmas served to construct accurate sundials for any location.

Hellenistic trigonometry

As is known, the first recorded use of trigonometry is derived from the works of Hipparchus, while later in his *Almagest* Ptolemy developed extended trigonometric calculations. Among other things, Ptolemy transformed Hipparchus's table of chords. First, since the length of the chord depends on (is proportional to)

the radius of the circle, Ptolemy constructed a more complete table of chords using a different large fixed radius. Second, his table had chords for angles increasing from 1/2 degree to 180 degrees in steps of 1/2 degree. Ptolemy had chosen a circle whose diameter and circumference were divided into 120 and 360 parts, respectively. Then, he calculated the chord length for every central angle up to 180 degrees in half-degree steps (Fig. 7).

This, in turn, enabled Ptolemy to calculate the chord lengths for the central angles that correspond to the equilateral and equiangular figures of the decagon (internal angles of 144°), hexagon (internal angles of 120°), pentagon (internal angles of 108°), square (internal angles of 90°), and triangle (internal angles of 120°).

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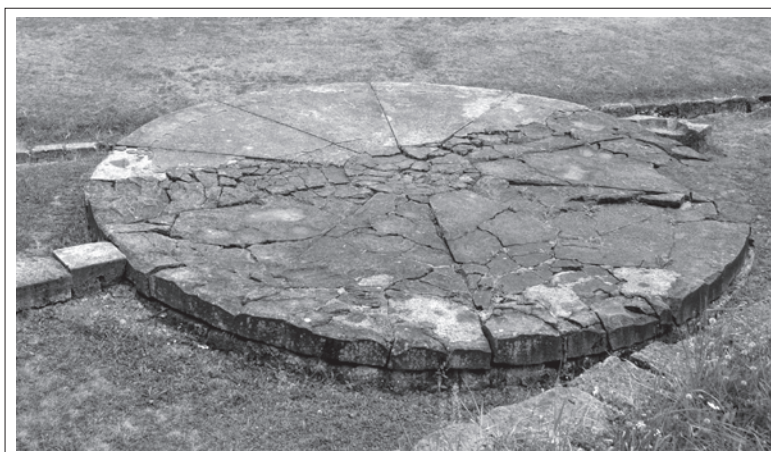
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Fig. 9. The Andesite Sun with the stone arrow discovered in 1978. Photograph by Daniel Baltat.
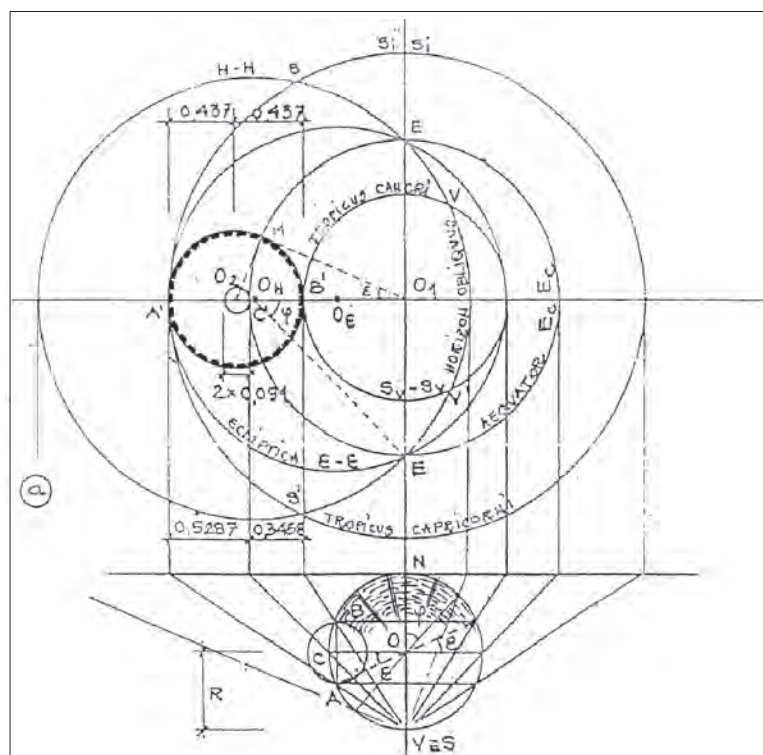

Fig. 10. The reconstruction of the supposed Hipparchus planispheric astrolabe.

Using the data in Figure 8 it can be proved that the chord lengths for the central angles are: 37° 4' 55" for a decagon, 60° for a hexagon and 70° 32' 3" for a pentagon.

"The Andesite Sun" at Sarmizegetusa Regia, Romania

Also placed on the Sacred Terrace, west of the Great Round Sanctuary (diam. = 30 m), is the Andesite Sun, a huge circular disk (diam. = 6.98 m), divided into ten slices of 36 degrees each. At 0.45 m from its edge

there are rectangular groves, some of them being "T" shaped.

Connected to one side of the disk is a long "arrow" made of limestone, reaching a total length of 9.55 m. The arrow of the Andesite Sun indicates the north. The idea is to consider the Andesite Sun to be a type of sundial.

The stereographic projection and the planispheric astrolabe

Attempts to project a sphere onto a plane resulted in the stereographic projection. Hipparchus (180-125 BC) is said to have formalized the projection as a method for solving complex astronomical problems without using spherical trigonometry. Although he refined projection theory and systematically used the 360° circle, he probably did not invent the astrolabe. The earliest evidence of the use of the stereographic projection in an artifact is due to Vitruvius (ca. 88 – ca. 26 BC). However, Hipparchus may have invented the planispheric astrolabe, used to define stellar (and solar) positions. The planispheric astrolabe is a two-dimensional model of the celestial sphere in relation to the earth, based on the assumption that the earth is at the centre of the universe. One side of the disk usually displays circles divided by different kinds of gradations, such as 360 degrees. It is only in Late Antiquity, when the zodiac was divided into twelve "signs" or thirty-six "decans", that a seasonal cycle of roughly 360 days could have corresponded to the signs and/or decans of the zodiac by subdividing each sign into thirty equal parts, or each decan into ten parts (Fig. 10).

The Results Obtained So Far, Including AN Interpretation

In the second part of the paper we will present some new hypotheses concerning the sundial called the "discus in planitia", ascribed by Vitruvius to Aristarchus, and the properties of the altar-sundial ("The Andesite Sun") at Sarmizegetusa Regia, Romania.

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SENOVĖS MATEMATINIŲ IR ASTRONOMINIŲ MATAVIMŲ SISTEMOS: PTOLEMĖJO SKAIČIAVIMAI STYGOMIS IR NAUJAS POŽIŪRIS Į „DISCUS IN PLANITIA" PRISKIRIAMĄ ARISTARCHUI. I DALIS

Florin Stanescu

Santrauka

Straipsnio tikslas – pristatyti naują hipotezę apie žymųjį andezitinį 7 m skersmens diską, kuris buvo rastas Pietų Karpatų kalnų regione, Rumunijoje, Senovės Dakų karalystės sostinėje, ant šventa laikytos terasos. Šiai konstrukcijai yra būdingi visi Saulės disko specifiniai bruožai: vietovės platuma φ, ekliptikos pasvirimas epochoje ε, šiaurės – pietų orientacija, todėl yra tikėtina, kad šis diskas yra tas pats „Discus in Planitia", kurį Vitruvijus priskyrė Aristarchui.

Vertė Audronė Bliujienė