THE PYTHAGOREAN GEOMETRY OF THE ATREUS TOMB AT MYCENAE

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Abstract

A geometrical analysis was performed using CAD (Computer Aided Design) tools on the plans of the nine Tholos Tombs of Mycenae and of the "Treasury of Atreus" in particular. Dedicated parameters were established in order to classify the main common geometrical features of the tombs. The analyses were based on a comparison between the geometrical proportions found on the plans and those of the Squaring Triads. It appears that Mycenaean architects made use of both Perfect (Pythagorean) and Quasi-Perfect combinations of integers. The Treasury of Atreus stands out by exhibiting all the major geometric proportions identifiable with those belonging to a series of Pythagorean Triads reported by Diophantus and known to the Mesopotamians. The unit of length for the Atreus Tholos Tomb coincides with the Lagash Gudea cubit of 0.496 m.

Key words: geometry, Pythagorean Triads, cubit, Mycenae, Dyophantus.

Introduction

The nine Tholos Tombs of Mycenae constitute a homogeneous group in terms of their spatial disposition (all are located on the western side near the Mycenae citadel), function (all are funerary) and architectonic typology (all have corbelled vault domes accessed from a Dromos). In addition, they were all built within the relatively short time span of two centuries. In the History of Architecture, the Treasury of Atreus represents the archetype of large corbelled vault domes and was the biggest in antiquity until the construction of Rome's Pantheon (1st century AD, 1½ millennia later).

To achieve the squaring (Ranieri 1997, 2002, 2005, 2006, 2007; Malgora 2000; Patanè 2006), three integers (a Squaring Triad) can be used, defining the lengths of sides and diagonals in terms of length-units. A reduced repertory for the present analysis is shown in Table 1. (For a wider repertory see Ranieri 1997).

Analyses and Results

The analyses were based on the association of the orthogonal forms as drawn on the plans with the proportions of the Squaring Triads (all plans are from Pelon 1976).

Three parameters were defined in order to classify the main orthogonal geometrical features:

- VA (Vault): the ratio of height to radius. VA= h/r.

- CH (Chamber): the ratio (Diameter+Stomion)/Diameter. CH=(F+s)/F

- DR (Dromos): the ratio of Dromos length to Diameter. DR = d/F

The results for the nine Tholoi are briefly summarized in table 2.

As shown in Fig.2 (with the exception of the Aegisthus Tomb for which CH = 3/W), parameter CH classifies the Tholoi into two groups:

a "√2-Group" (Cyclopean, Epano Phournos, Panagia, Clytemnestra) and

a "D-Group" (Kato Phournos, Lion, Genii, Atreus).

In each group, there is modularity despite the diversity of sizes.

Table 2 also suggests that the vault heights were all established following the proportion 15/8 of the P-triad M=8-15-17.

For the Treasury of Atreus, as can be seen in Fig.3, all the most significant geometrical proportions appear to have been governed by Pythagorean Triads, namely D (CH), W (DR), M (VA), and GA. This is of special interest, because these triads are the first four terms of a series of P-triads that can be generated using the numerical algorithm reported by Diophantus (3rd century AD) in Book II, Problem 8 of his Arithmetica[1]. This algorithm was known to the Mesopotamians[2] in Mycenaean times long before Diophantus.

Units of length can be derived from the analyses. For the Treasury of Atreus, a "cubit-like" length-unit of 0.496 m ± 0.004 resulted. This value corresponds (well within the ± 0.004) to the values of known Mesopota-

[1] Given M and N mutually prime with M >N of different parities, if $A = M^2 - N^2$, $B = 2\,M\,N$, $C = M^2 + N^2$ then $A^2 + B^2 = C^2$.

[2] Mesopotamians were highly acquainted with P-triads, as demonstrated by the clay tablet N° 322 in the Plimpton collection at Columbia University (Neugebauer 1957).

Table 1. Triads related to the nine Tholoi
From left: Symbol, Integers, proportion r = B/A

Symbol	A	B	C	r=B/A	Symbol	A	B	C	r=B/A	Symbol	A	B	C	r=B/A
Q	7	7	10	1	4/W	24	28	37	1.6667	W	5	12	13	2.4
Q	12	12	17	1	MA	4	7	8	1.75	WB	10	25	27	2.5
3/W	20	25	32	1.25	M	8	15	17	1.875	2D	6	16	17	2.6667
D	3	4	5	1.3333	MC	12	23	26	1.9167	G	12	35	37	2.9167
√2	12	17	21	1.4167	2Q	12	24	27	2	3Q	6	18	19	3
L	10	15	18	1.5	2Q	13	26	29	2	GA	7	24	25	3.4287

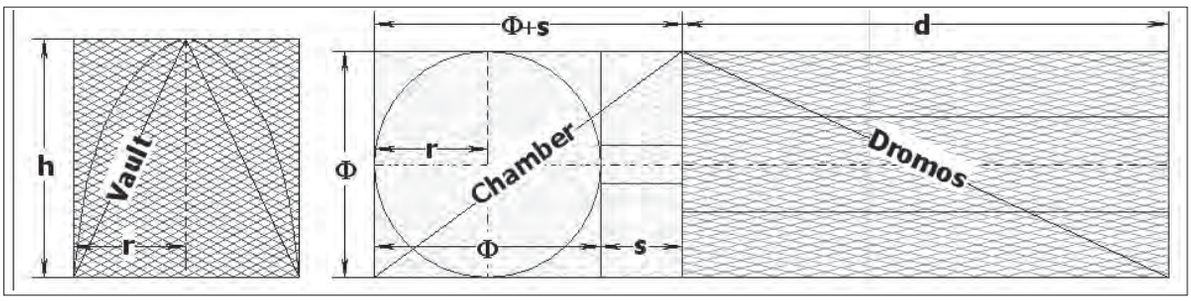

Fig. 1. Rectangular Parameters for a geometrical classification of Mycenaean Tholos Tombs.

Table 2. The resulting association of the Geometrical Parameters with Squaring Triads

THOLOS	VA	CH	DR	Other Triads
Cyclopean	?	√2	MC	Q, W
Epano Phournos	M	√2	2Q	Q, W
Panagia	M	√2	MA	Q, W, MC,G, L
Clytemnestra	M	√2	2D	Q, W, D
Kato Phournos	?	D	4/W	Q, 3Q, √2, WB, 2D, MC
Lion	?	D	19/12	Q, 3Q, M, G
Genii	M	D	2Q	Q, 3Q, 2D, L
Atreus	M	D	W	Q, 3Q, 2D, GA
Aegisthus	M	3/W	4/W	3Q, 3/W, M, MA

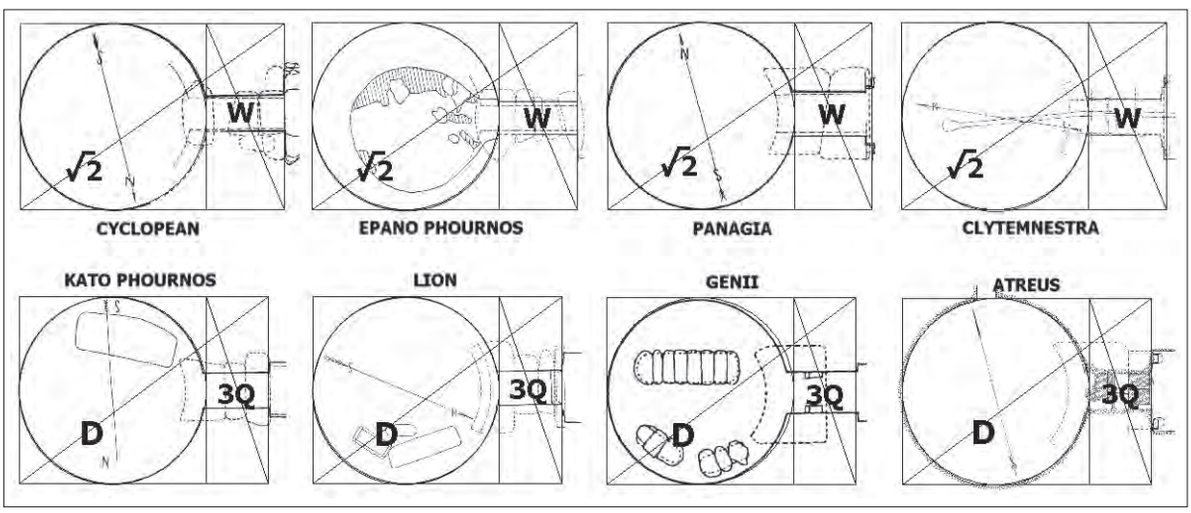

Fig. 2. The tholoi as classified using the CH "Chamber" geometrical parameter.

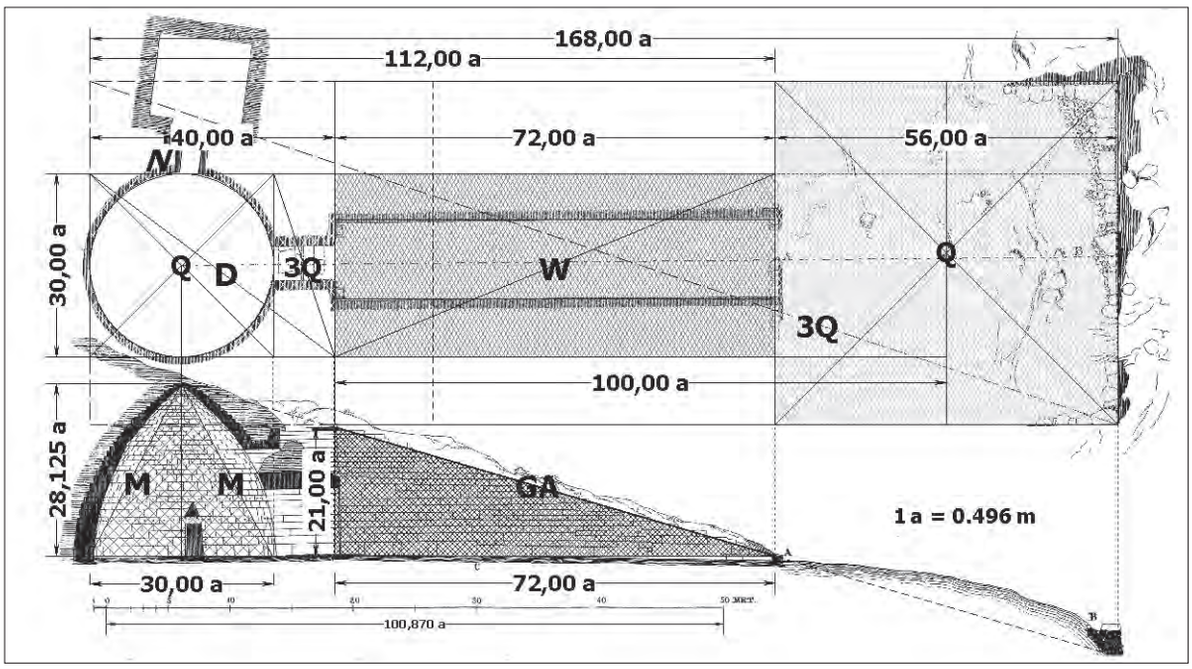

Fig. 3. The Pythagorean geometry of the Treasury of Atreus.

mian cubits: the Assyrian cubit of 0.494 m; the Sumerian cubit of 0.500 m; and the Gudea of Lagash statue cubit of 0.496 m.

Conclusions

It appears that Mycenaean architects made use of both Perfect (Pythagorean) and Quasi-Perfect combinations of integers. The Treasury of Atreus stands out by having all the major geometric proportions identifiable with those of the series of Pythagorean Triads reported by Diophantus and known to the Mesopotamians. The length-unit for the Atreus Tholos Tomb coincides with the Lagash Gudea cubit of 0.496 m.

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ATRĖJO TOLAS MIKĖNUOSE IR PITAGORIEČIŲ GEOMETRIJA

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Santrauka

Naudojant kompiuterinę automatizuoto projektavimo sistemą CAD (Computer Aided Design) buvo atlikta devynių laidojimo rūsių – tolų ir atskirai kapavietės, vadinamos „Atrėjo lobynu", geometrinė analizė. Siekiant klasifikuoti tyrinėjamas kapavietes, buvo išskirti bendri geometriniai tyrinėjamų laidojimo rūsių parametrai. Analizės pagrindas buvo geometrinių laidojimo rūsių proporcijų lyginimas pagal kompiuterine analize sudarytus planus su Pitagoro skaičių trejetais. Atrodo, kad Mikėnų architektai statiems kampams nustatyti naudojo tiek tikslius Pitagoro skaičių trejetus, tiek jų

artinius. „Atrėjo lobynas“ turi visas svarbiausias aiškiai atpažįstamas geometrines proporcijas, aprašytas Diofanto bei žinotas Mesopotamijoje ir būdingas Pitagoro skaičių trejetų geometrijai. „Atrėjo lobyno“ laidojimo rūsyje naudotas ilgio matas sutampa su Lagašo valdovo Gudėjo (Gudea) uolektimi – 0,496 m.

Vertė Audronė Bliujienė