A GEOMETRICAL ANALYSIS OF MESOAMERICAN PRE-HISPANIC ARCHITECTURE: SQUARING TRIADS, NUMBERS, LENGTH UNITS AND THE CALENDAR

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Abstract

This work extends to Mesoamerica researches on archaic geometry previously performed on architectures of the "Old World". CAD (Computer Aided Design) tools are used to fit theoretical forms to spatial segments on archaeological plans. The analyses provide quantitative information on the forms, on the numbers involved and on the units of length used. Everywhere, numbers appear mainly to be multiples of 9, 10 and 13, rather than pertaining to Squaring Triads, although these are nonetheless attested in all regions. Three units of length are found: a "Teotihuacán unit" $t \approx 0.58$ m, a "Monte Albán unit" $ma \approx$ 0.50 m and a "Chichen Itza unit" $ci \approx 0.66$ m. Units t and ma clearly appear to be linked to the distances between the main monuments, associated with the Tzolkin calendrical numbers 13-20 at Teotihuacán and with the Haab calendrical numbers 18-20 at Monte Albán.

Key words: geometry, Squaring Triads, length-units, calendar, Mesoamerica, Teotihuacán, Monte Alban, Chichen Itza.

Introduction

The repertory of distinct geometrical forms identifiable on archaeological plans consists mainly of right-angled shapes and circles.

In the vast majority of cases, right-angled shapes prevail and show a remarkable perfection. Among the supposed methodologies that the ancients may have used to achieve the squaring, the most interesting is the one in which three appropriate integer numbers (a Squaring Triad or Triple) define the lengths of the sides and diagonals in terms of given units of length. This approach has proved successful in decoding geometric arrangements of relevant structures in the "Old World" (Ranieri 1997, 2002, 2005, 2006, 2007, 2008; Malgora 2000; Patanè 2006). A suitable repertory of Squaring Triads can be found in Table 2.

Circular forms can be predominant in certain contexts (as for the Nuraghic culture) but are also present in rectangle-dominated contexts. Of importance for the present work is the "Circle-Square-Circle" (CQC) composition: two concentric circles, the smaller of which is inscribed within, and the larger circumscribed to, a square. In CQCs the dimensions of the diameters are those of the sides and diagonal of the square (Fig.1a) so that the circular dimensions can be deduced in their turn from the squaring numbers of the square (first four entries in Table 2). A study of the occurrence of CQC patterns in the geometry of ancient structures is under way and will be presented elsewhere.

Methodological Approach

Even for a single monument – in my experience - a through analysis would require about one year on average. Given that there are hundreds of Mesoamerican sites and monuments, analyses of even only a hundred of them would therefore take a century. Therefore I opted for a more practicable (3-4 years' work) "large-number" approach with less detailed analyses (only providing "good-fits" rather than best-fits) on about 150 monuments in the hope that statistics would eventually help to explain major geometric features (if any existed) common to all Mesoamerican architecture. As we shall see, the results have been more than encouraging with unexpected discoveries of undeniable importance. With few exceptions, plans were taken from Marquina (1951), starting with plans of more than 200 monuments altogether. About 75% were considered eligible for CAD analysis.

"Good-fits"

For the Squaring Triads in the repertory, the differences between two nearby proportion values (r = B/A) can be small. For this reason, "triadic" interpretations may be of low reliability with uncertainties on average greater than 3%. On the other hand (and unlike what had been experienced with "Old World" artefacts), Triads, although clearly present, do not appear to be the principal characters in the numerical repertory of Mesoamerican architecture. It appears rather that the

A Geometrical Analysis of
Mesoamerican Pre-Hispanic
Architecture: Squaring Triads,
Numbers, Length Units
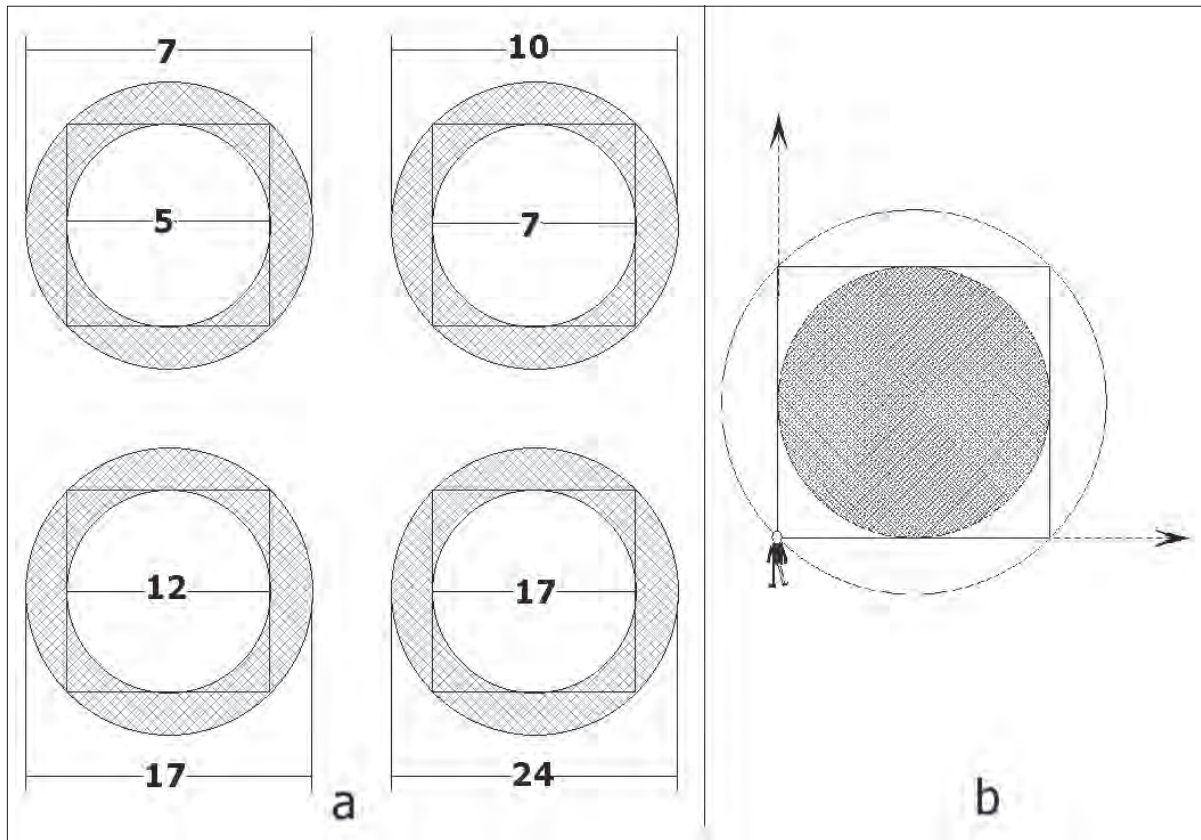and the Calendar

**MARCELLO
RANIERI**

Fig. 1. In CQCs the area of the circular ring equals the area of the inner circle or conversely the area of the outer circle doubles the area of the inner one. From any point on the outer circumference the lines of sight tangent to the inner circumference are orthogonal (Fig.1b).

attention of the Mesoamericans focused on calendrical numbers (13, 18 and 20) and their multiples. A class of "calendrical integers" was therefore introduced, which includes the multiples of 9 (=18/2), of 10 (=20/2) and of 13.

In the CAD analysis, for a single length to be expressed in terms of an integer number of length-units, the uncertainty needs to be better (lower) than the length-unit itself: otherwise nearby integers are indiscernible. This difficulty can often be satisfactorily overcome when fitting a plurality of linear segments if it can be safely hypothesized that a unique length-unit was used. Care was taken not to mix segments belonging to different construction phases. Three parameters were introduced to control the goodness of the fits.

- Parameter "sc.un.": the metric scale uncertainty, CAD-measured in cm.
- This is often the major cause of error in the evaluation of the length-unit.
- Parameter "grafic": the uncertainty represented by the lines' thicknesses (CAD-measured in cm on an average line). This is the main cause of error in fitting forms and segment lengths with integer numbers. Forms and integers are accepted when

their lines (or CAD measuring-tool lines) coincide with the plans' lines within the thicknesses or, for a small percentage (<5%) on a single monument, at most 1 "grafic" away.
- Parameter "Rcal": the number of lengths that are expressed with a calendrical integer in a set of N CAD-measures; to be compared with the probability of any randomly chosen integer being calendrical, represented by a density function which ranges around an expected value of 27%.

The search for length-units was for "cubit-like" values (40 – 70 cm).

Analyses and Results

Since the results are mostly of a graphical nature, ideally they should all be presented illustrated by plates and figures. However this would by far exceed the number of pages allowed for these Proceedings. Hopefully, the whole will better appear in a monograph in the future. The results are intended to be "virtual" in the sense that, while they do apply to the plans, they may not correspond to the real structures, at least pending verification of the correspondences between plans and monuments.
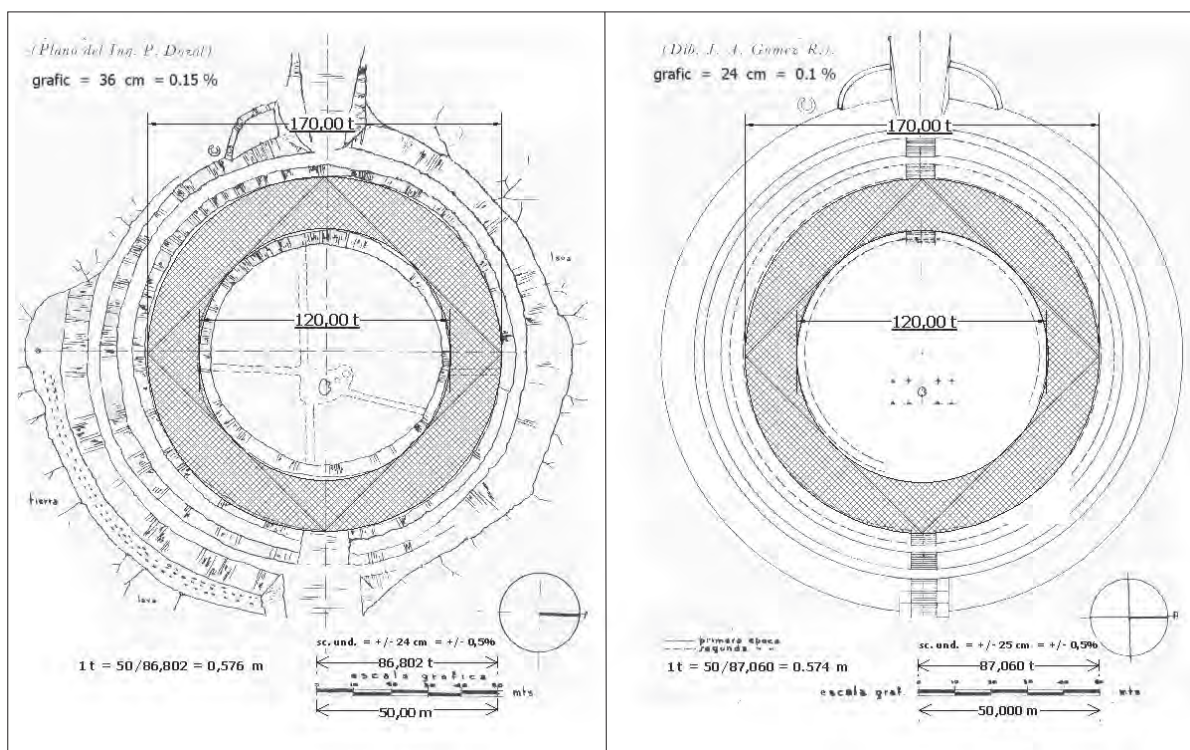
Fig. 2. Cuicuilco: A 12-17 CQC fitting the circumferences outlining the terrace at 3rd level.

Cuicuilco

The first monument encountered in Marquina's book is the "monumento de planta circular" of Cuicuilco: a huge four-level "circular pyramid" some 135 m in diameter at ground level and about 30 m in height. In the published plans[1] (from the 1920s after the excavation and from the 1950s after restoration) a CQC (12x10-17x10) was fitted onto the circumferences that outline the main terrace at the third level (Fig.2). Among the possible values for "cubit-like" units from multipliers 14, 13, 12, 11, 10 and 9, t = 0.575 m (multiplier 10) was chosen because it fitted better the other dimensions of the monument.[2] From the two plans the values 0.576 ±0.002 m and 0.574 ±0.002 m yield an average t = 0.575 ±0.002 m. As we shall see, t suits a large number of other Mesoamerican monuments, Teotihuacán included.

Teotihuacán

The second site is much-renowned Teotihuacán. There is a general plan of the ceremonial centre along the main avenue, together with plans of the Pyramids of the Sun and Moon, the Citadel, and of other monuments (Tepantitla, Tetitla, Atatelco, the Viking group, and Xolalpan).

The Pyramid of the Sun, the Pyramid of the Moon, and the Citadel

In the plan (Fig.3), the Pyramid of the Sun seems to form part of a segmented environment whose main rectangular CAD-measured dimensions are 459.3 ±0.65 m and 345.4 ±0.65 m. Their ratio yields r = 1.330 ±0.003 which, within 0.3%, equals the 4/3 = 1.333... ratio of P-triad D=3-4-5. The fit with a rectangle 600x800 (multiplier 200) yields a t unit of 0.574 ±0.006 m, the very same as that found for Cuicuilco. Because of this exact coincidence I retained the value of 0.575 m and tried it out on the plan of the Pyramid of the Sun itself. With sc.un. = 81 cm and grafic =101 cm, the integers in Fig.3 were those that more closely fitted the CAD-measured lengths. It is worth noting the values of 390 and 400 for the base dimensions (390=13x30 and 400 =20x20) and the Rcal value =100%.

For the Pyramid of the Moon, (sc.un. 41 cm, grafic 28 cm) a D proportion was easily found at the second level (Fig.4). Scaling the measurements in t units and with a multiplier of 40, a fit of 120x160 could be obtained. The other numbers followed as a consequence yielding Rcal = 84%. Note that the dimensions 108x45 of the

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[1] With grafic = 36 and 24 cm respectively and sc.un.=25 cm for both plans.

[2] Other length-units may derive either from the quite imprecise (squaring more than 1° away from 90°) CQC combinations 5-7, 7-10 or from 17-24 and their multiples. The 12-17 combinations were considered first, being more precise than 5-7 or 7-12 and allowing the same precision (only 12' away from 90°) as combination 17-24 but with smaller numbers.

A Geometrical Analysis of
Mesoamerican Pre-Hispanic
Architecture: Squaring Triads,
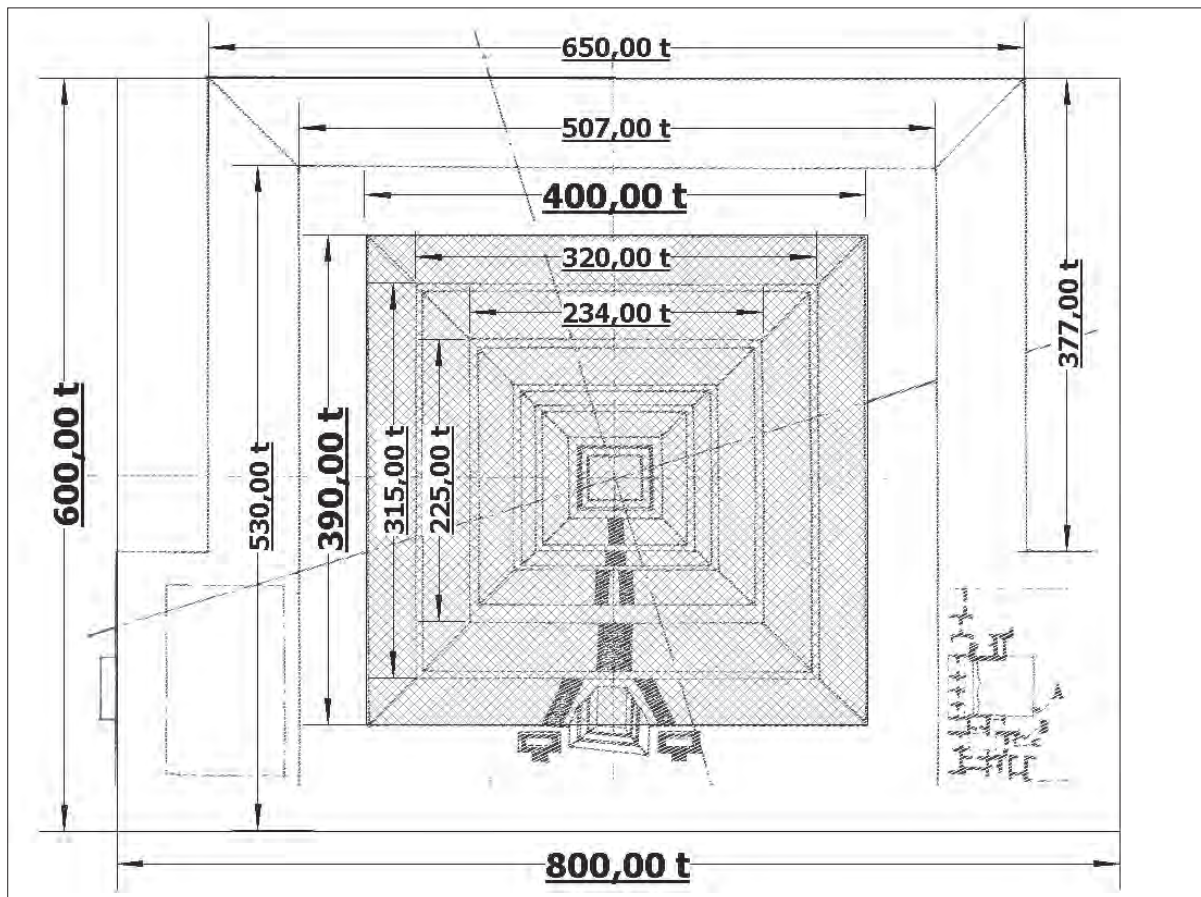Numbers, Length Units
and the Calendar

**MARCELLO
RANIERI**

Fig. 3. Teotihuacán: Pyramid of the Sun. Measurements in **t** units of 0.575 m.

lower stairs' basement correspond to the P-triad W=5-12-13 through a (calendrical) multiplier 9.

The plan of the Citadel bears a regrettable discrepancy between the hand-drawn measures and the corresponding CAD measurements based on the metric scale, so that two different units with large uncertainties (±0.033 m) resulted. Averaging yields 0.563 ±0.033 m, **t** being well within this range.

The Street of the Dead

The distances along the Street of the Dead, between the centres of the Pyramid of the Sun and the Pyramid of the Moon, and between the centres of the Pyramid of the Sun and the Pyramid of the Feathered Serpent, were measured by CAD both in metres and in **t** units. The results are shown Fig.5. Clearly, **t** values to within less than 0.04% (grafic 82 cm, sc.un. 150 cm) coincide with 2000 **t** and 1300 **t**, i.e. 100 times the very same Tzolkin calendrical numbers 13 and 20. Such a close correspondence together with the fact that, as shown below, the same **t** unit fits all the 13+2 Teotihuacán and Cuicuilco plans, makes it difficult to consider these circumstances altogether fortuitous. Rather, in my view,

the result shows that at Teotihuacán space (distances between main monuments) and time (the calendar) were firmly and inexorably linked together through numerology, very likely since the first establishment of the ceremonial centre. Given that Teotihuacán is acknowledged by most scholars as the most sacred place of the Mexican prehispanic cultures, the result is of special importance and certainly merits future verifications and confirmations.

Tepantitla, Xolalpan, Viking group, Tetitla, Atatelco, Quetzalpapalotl Palace (Plans for the Quetzalpapalotl Palace were taken from Acosta (1964))

The situation is favourable for the plans of these groups, mainly because of the small dimensions of the artefacts. As an example, Fig.6 shows the fits for Tepantitla and Xolalpan. Note the high Rcal (35% and 73%) and the presence of integers related to Q-triads (Q, 2Q, 3/W). Comparable results were obtained for the Viking group, Tetitla, Atatelco and the Quetzalpapalotl palace: fits were obtained for all of them in t units within the thickness of the lines.

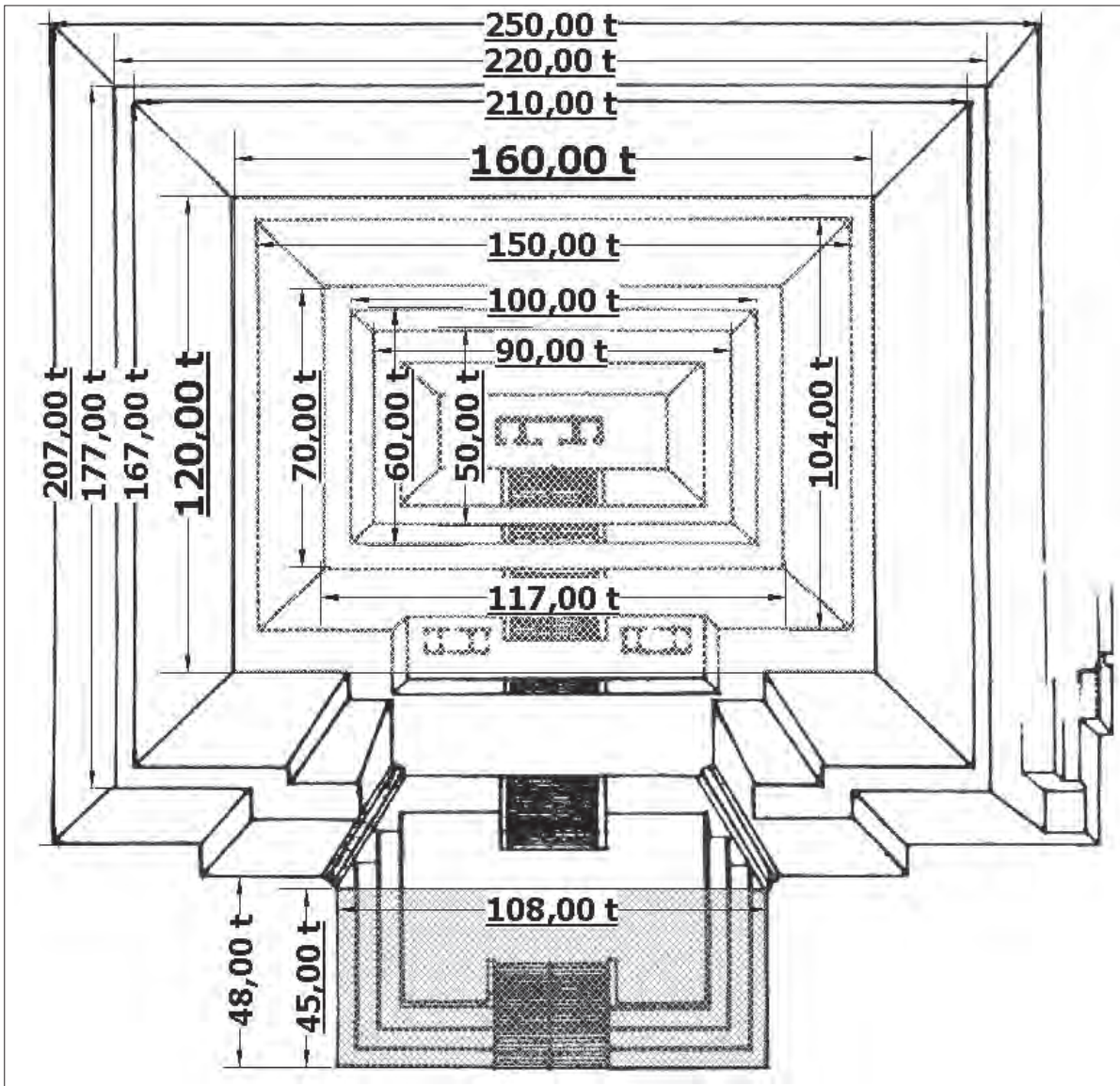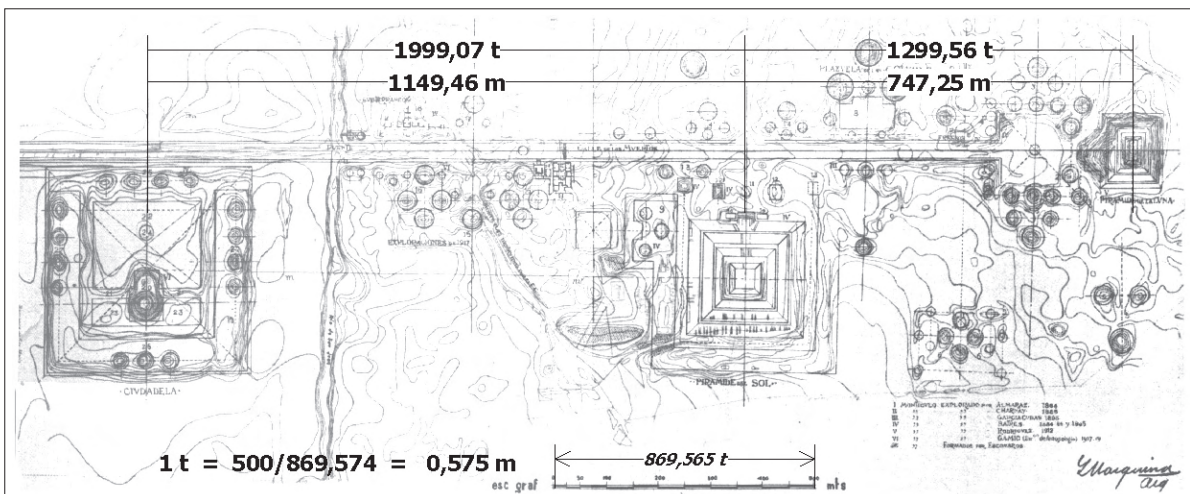Fig. 4. Teotihuacán: Pyramid of the Moon. Measurements in **t** units of 0.575 m.

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Fig. 5. Teotihuacán: measurements in metres and in **t** units along the Street of the Dead.

A Geometrical Analysis of
Mesoamerican Pre-Hispanic
Architecture: Squaring Triads,
Numbers, Length Units
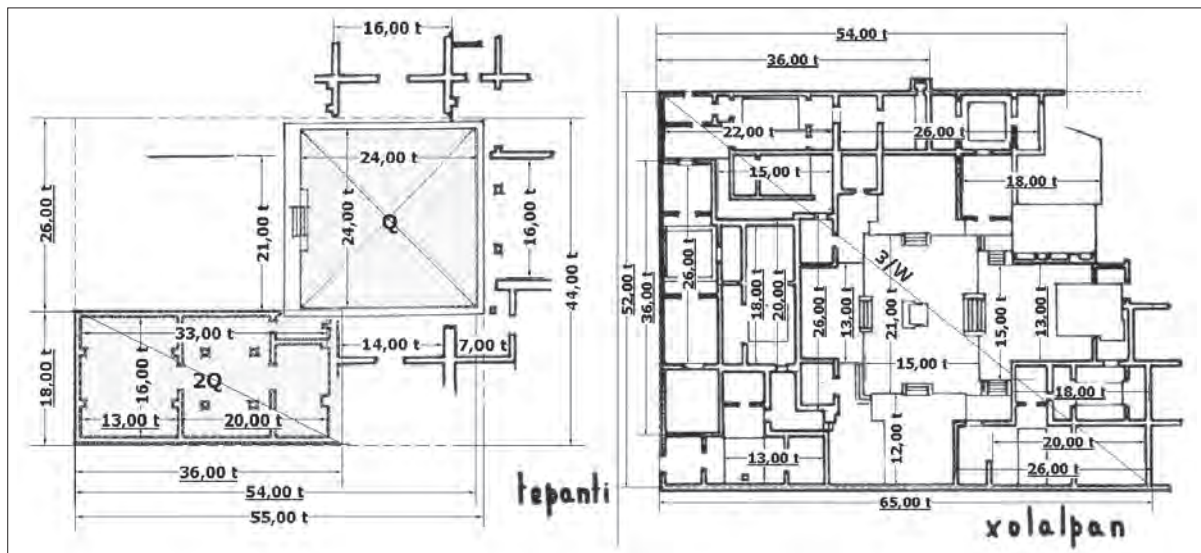and the Calendar

**MARCELLO
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Fig. 6. Teotihuacán: Tepantitla and Xolalpan. Results in **t** units.

Sites in nearby regions

For Cholula, Xochicalco, Tula, Calixtlahuaca, Tenayuca, Tenochtitlan, Teopanzolco and Tzintzuntzan, 21 plans were analysed. Table 1 summarizes the results including Cuicuilco and Teotihuacán for a total of 35 monuments and clearly shows that **t** units allow good fits for the vast majority of the monuments in the region with the average calendrical content a good deal above the expected 27%.

Monte Albán

Monte Albán is another very sacred place in ancient Mesoamerica. The site is structured with its three major monuments aligned along one axis (North-South), much like Teotihuacán, but on a smaller scale. Four plans were eligible: the "Los Danzantes" building, the "Pelota" court, Building M, and the general plan of the site.

The general plan (in the plan the metric scale was in error. The correct one has been derived from Sterling, 1968)

By analogy with Teotihuacán, the first things to be measured were the distances between the major monuments. The CAD-measures (Fig.7) were 201.8 m and 181.6 m. Their ratio is 201.8/181.6 = 1.1112..., which is 20/18 = 1.1111... to within 0.01%. Again, as for Teotihuacán, distances appear to represent calendrical numbers: 18 and 20 of the Haab calendar in this instance. With a multiplier of 10, a length unit **mu** = 1.009 m was obtained and half of it (which is in the

cubit-range) was adopted as a "Monte Albán unit" **ma** = 0.504 ±0.007 m.

"Los Danzantes", the "Pelota", and Building M

Of the three analysed plans, two (Rcal <27%) fitted with **ma** units: 0.504 m for "Los Danzantes" and 0.499 m for building M. The "Pelota" fitted instead in **t** units (0.575 m) with Rcal = 60%.

On the west coast of the Gulf of Mexico: Tajin, Cempoala and Misantla

At Tajin, all 9 plans fitted with **ma** units of 0.499 m: The Pyramid (Fig.8), Chico A,B,Q, Buildings 1, 2, 5, and the Los Nichos Pyramid; Chico C 0.509 m. All the Rcal were between 50% and 63%, except at Chico Q (9%).

At Cempoala, 7 plans out of 8 fitted with **ma** units: Las Chimeneas 0.487 m (Rcal 25%), Las Caritas 0.500 m (Rcal 51%), Montecuzuma House 0.499 m (Rcal 36%), Dios Del Aire Temple (Fig.10) 0.499 m (Rcal 40%), The Plaza 0.499 m (Rcal 68%), Las Chimeneas 0.499 m (Rcal 83%), and Dios Del Aire (Substructure) 0.506 m (Rcal 38%). The value for the Grand Pyramid was 0.463 m (close to 4/5 **t**.) (Rcal 42%).

At Misantla there were 2 plans: Building F fitted with **ma** = 0.499 m (Rcal 46%) while Building A fitted with a **t**-like unit of 0.564 m (Rcal 75%).

Summarizing, of 18 monuments from 3 analysed sites, 17 share the same **ma** unit (the exception being Building A at Misantla).

Table 1. Results for sites in regions near Teotihuacán

	Sites and Monuments	grafic (cm)	scale (m)	sc.un. (cm)	sc.un. (%)	Length unit (m)							Rcal
	Cuicuilco (2)												
1	1	±36	50	±24	0.48		0.574						-
2	2	±24	50	±25	0.48				0.576				-
	Teotihuacan(12)												
3	Pyr.Sol	±63	100	±66	0.66		0.574						100%
4	Pyr.Luna	±66	100	±41	0.41			0.575					91%
5	Citadel	±188	100	±65	0.65	0.563							100%
6	Avenue	±97	500	±199	0.40			0.575					100%
7	Tepantiltla	±12	30	±12	0.40			0.575					40%
8	Viking	±16	30	±20	0.67			0.575					43%
9	Tetitla a	±12	30	±11	0.37			0.575					46%
10	Tetitla b	±12	30	±11	0.37			0.575					78%
11	Atatelco	±9	30	±12	0.40			0.575					64%
12	Xolalpan	±10	30	±12	0.40			0.575					73%
13	Quetz.p.	±5	25	±4	0.16			0.575					30%
14	Quetz.p	±5	4	±4	1.00			0.575					32%
	Cholula (1)												
15	Pyramid	±35	100	±45	0.45			0.575					60%
	Xochicalco (2)												
16	Mon. Descub.	±15	10	±15	1.50				0.576				30%
17	Pelota	±18	50	±23	0.46					0.578			62%
	Tula (3)												
18	Ed. de Hidalgo	±12	20	±18	0.90				0.576				90%
19	Edificio 1	±6	10	±7	0.70		0.574						
20	Pelota	±11	20	±25	1.25						0.580		64%
	Tenayuca (7)												
21	Pyr. 1st epoch	±26	30	±35	1.17			0.575					28%
22	Pyr. 2nd epoch	±31	30	±29	0.97			0.575					65%
23	Pyr. 3rd epoch	±30	30	±21	0.70			0.575					47%
24	Pyr. 4th epoch	±30	30	±32	1.07			0.575					32%
25	Pyr. 5th epoch	±12	30	±20	0.67			0.575					39%
26	Pyr. 6th epoch	±16	30	±23	0.77			0.575					33%
27	Pyr. all epochs	±31	30	±15	0.50	0.568							56%
	Tenochtitlan(1)												
28	General plan	±101	130	±97	0.75							0.589	52%
	Tepoztlan (1)												
29	Mon.de Tepozteco	±9	10	±7	0.70			0.575					24%
	Teopanzolco (1)												
30	Pyramid	±15	10	±13	1.30	0.561							35%
	Calixtlahuaca (4)												
31	Circ.Tem.1st epoch	±7	10	±13	1.30			0.575					46%
32	Circ.Tem.2nd epoch	±7	10	±13	1.30			0.575					56%
33	Circ.Tem.3rd epoch	±7	10	±13	1.30			0.575					30%
34	Circ.Tem.4th epoch	±7	10	±13	1.30			0.575					47%
	Tzintzuntzan (1)												
35	Yacata	±26	20	±31	1.55			0.575					70%

A Geometrical Analysis of
Mesoamerican Pre-Hispanic
Architecture: Squaring Triads,
Numbers, Length Units
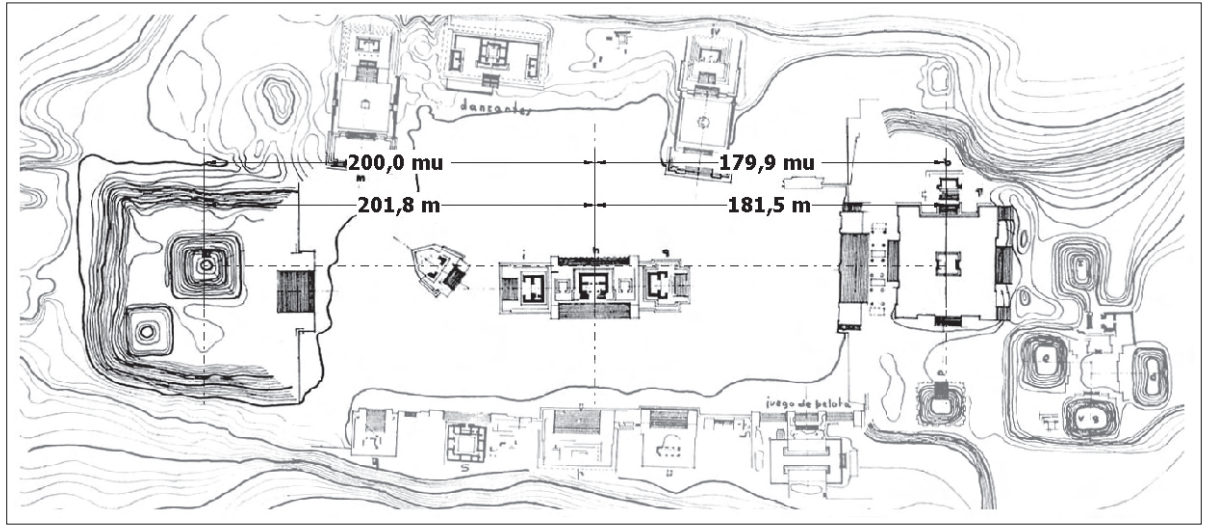and the Calendar

**MARCELLO
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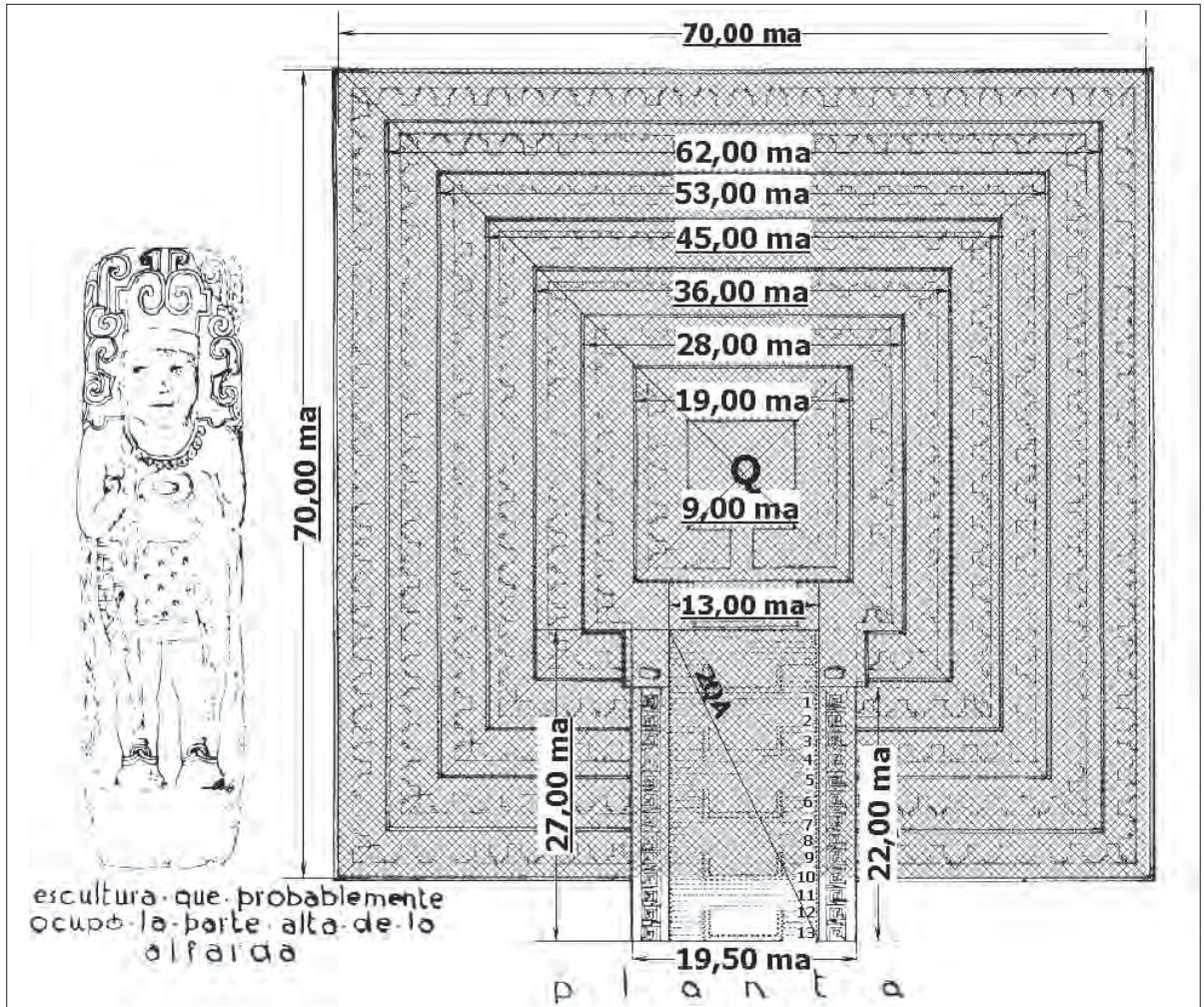Fig. 7. Monte Albán, CAD measurements in metres and **mu** units.
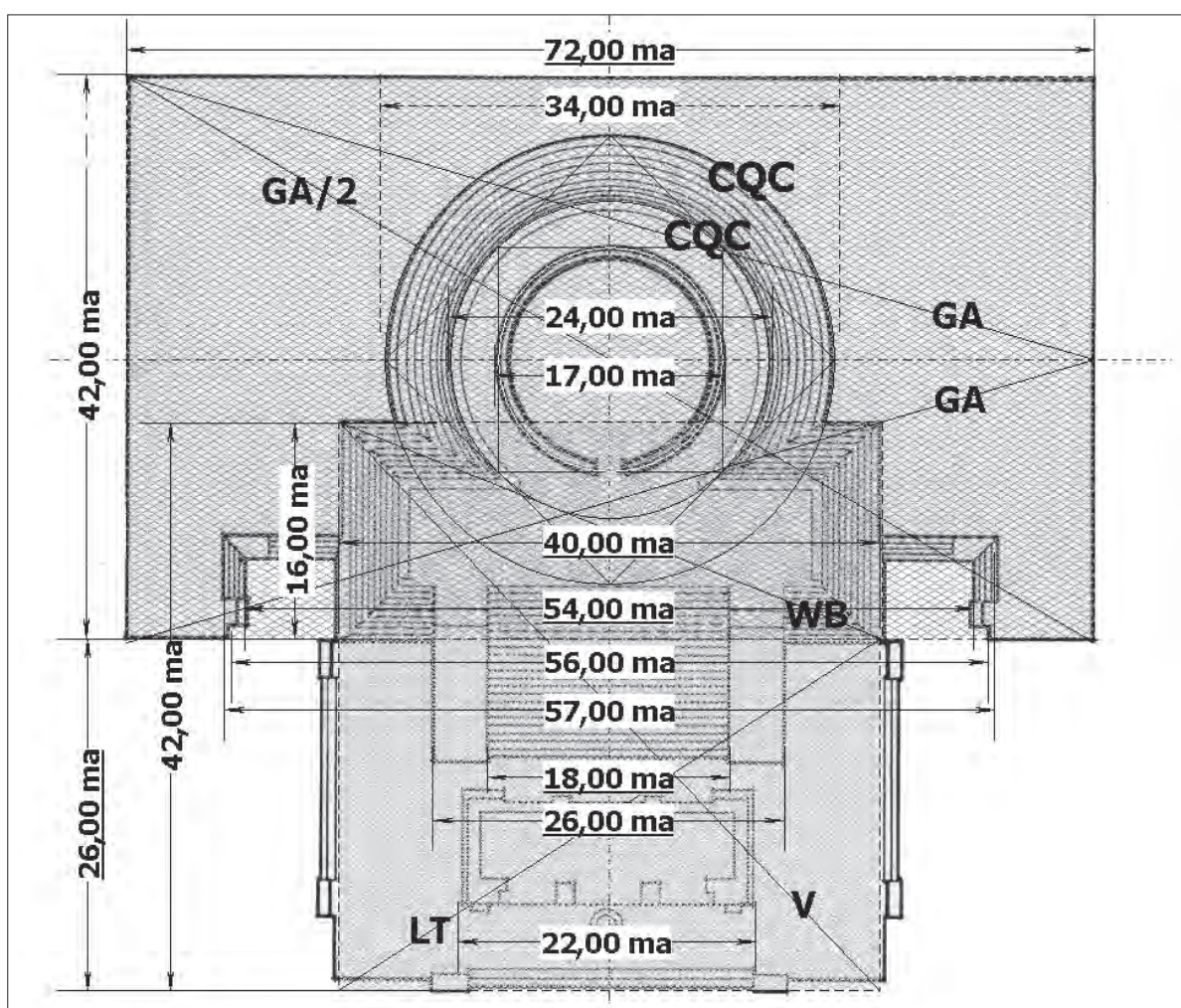

Fig. 8. Tajin Pyramid in **ma**-units of 0.499 m**.**

Fig. 9. Cempoala: Geometry of the "Templo del Dios del Aire". Measures in **ma** units of 0.499 m The presence of P-Triads, Q-triads and CQCs is noteworthy.

Other Oaxaca sites: Yucunudahui, Mitla, Montenegro

Five monuments were analysed, and all fitted with t = 0.575 m and high Rcal values: Yucunudahui's "Pelota" Rcal 43%, Tlaloch Temple and "Mogote Grande" Temple Rcal 50%; Mitla's "Las Columnas" Rcal 36% and Montenegro's Plan Rcal 60%. The t unit appears to be predominant in Oaxaca. This includes Monte Albán, although there, as expected, **ma** units were also found.

Yucatan

49 monuments (at 18 sites) were analysed and the majority gave high Rcal values. Alongside t units and **ma** units, in the Yucatan another unit ci = 0.665 ±0.015 m appears well documented, at Chichen Itza in particular (possibly two in Xelha and one in Cakacal where a unit of 0.206 m, about 1/3 **ci**, was found).

"Chichen Itza" **ci** unit

20 monuments (from 10 sites) fitted with units in the range 0.65-0.68 m:

Chichen Itza (5)   (2x) 0.660 m, 0.665 m, (2x) 0.656 m
Itzamal (1)   0.681 m
Chacmultun (3)   (3x) 0.660 m
Acanceh (1)   0.666 m
Edzna (1)   0.662 m
Coba (1)   0.660 m
Tulum (2)   0.651 m, 0.660 m
Kucican (1)   0.669 m
Nohoch Mul (2)   0.672 m, 0.677 m
Macancox (3)   0.666 m, 0.672 m, 0.677 m

Another five monuments from Uxmal (2), Chacmultun, Kabah and Coba yielded units in the range 0.62–0.64 m. (0.63 ±0.01), not far from **ci** (-5%).

A Geometrical Analysis of
Mesoamerican Pre-Hispanic
Architecture: Squaring Triads,
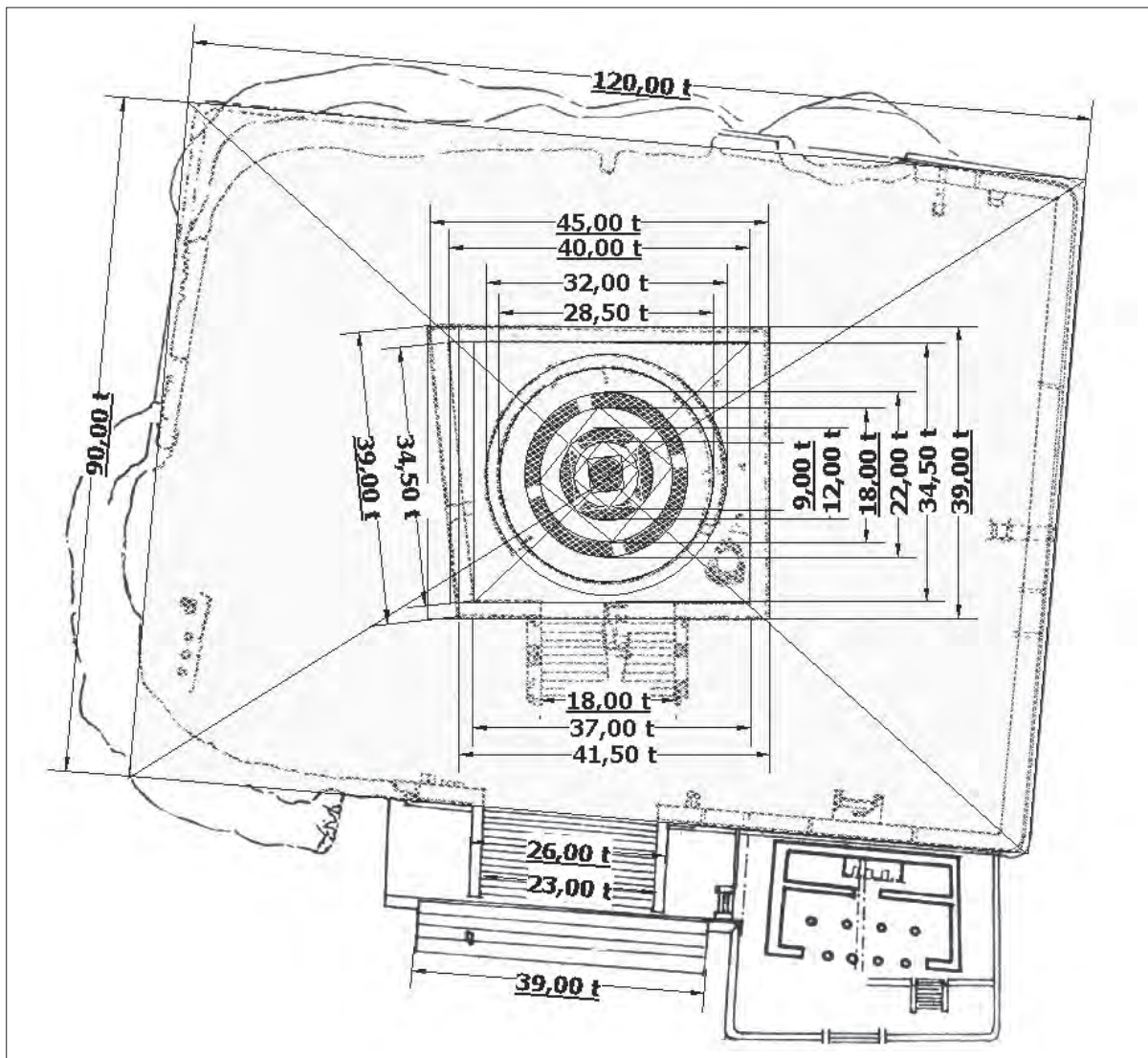Numbers, Length Units
and the Calendar

**MARCELLO
RANIERI**

Fig. 10. Chichen Itza: The Caracol in units of 0.57 m, close to Teotihuacán's unit **t** = 0.575 m.

t, ma and other units

14 monuments (from 4 sites) fitted with **t**: Uxmal (9), Chichen Itza (3), Acanceh (1), and Kucican (1).

3 fitted with **ma**: Meco 0.496 m, Zayil 0.501 m and Cozumel (3/2 **ma**).

Small units were found in Labna (3) 0.362 m, Tulum 0.391 m, Cacakal and Xelha (2) 0.206 m.

Chiapas

27 plans were analyzed from 7 sites in Chiapas.

25 of these (from 6 sites) fitted in **t**-units (with considerable dispersion): Yaxchilan (Rcal 64-73%): 12 x 0.575 m; Tenam (Rcal 30%): 0.575 m, 3 x 0.575 m; Tonina (Rcal 100%): 2 x 0.575 m; S. Elena Poco Uinic

(Rcal 73%): 0.575 m; Agua Escondida (Rcal 50%): 0.575 m; and Palenque (Rcal 45-72%): 0.575 m, 0.573 m, 0.570 m, 0.568 m, 2 x 0.569 m. The other two were Piedras Negras (Rcal 50%): 0.439 m; and Palenque (Rcal.29%): 0.504 m (**ma**).

With only two exceptions out of 27 analyses, it appears that the sites in the Chiapas region share **t** units.

Peten and nearby regions

We analysed 4 sites for a total of 13 monuments, mostly interpretable in terms of **ci**-like units and **t**-like units. Tikal (Rcal 53-76%): 0.578 m, 0.563 m and (2) 0.569 m; Copan (Rcal 50-75%): 2 with 0.541 m and 2 with 0.658 m; San Jose (Rcal 35-100%): 4 with 0,674 m; and Zacpeten (Rcal 50%): 0.638 m.

Table 2. Squaring Triads. From left: Triad symbol; Integers A,B,C; r=B/A; sites where they are found

	A	B	C	r	sites
Q	5	5	7	1	Tajin, Acanceh(2), Tikal.
Q	7	7	10	1	Viking. Quetzalpapalotl Palace, Tajin, Cempoala, Macanxoc.
Q	12	12	17	1	Tepantitla, Quetzalpapalotl Palace, Xochicalco, Tajin(2), Cempoala(3), Agua Escondida, Chichen Itza, Nakum(2).
V	**20**	**21**	**29**	**1.05**	Cempoala.
VC1	10	11	15	1.1	Izamal, Tikal.
VC2	18	20	27	1.111	Teotihuacan Citadel. Cempoala, Chichen Itza, Chacmultun, Tulum, Coba, Macanxoc, Xelha+Cacakal(2),
S	8	9	12	1.125	Viking. Atatelco, Xochicalco, Yaxchilan, Chacmultun, Cozumel.
SA	13	15	20	1.154	Yaxchilan, S.Elena P.U., Chichen Itza, Acanceh, Coba, Tikal.
SB	19	22	29	1.158	Tulum.
SC	11	13	17	1.182	Xelha+Cacakal.
W/2	25	30	39	1.2	Viking, Tenochtitlan(2), Misantla, Uxmal, Izamal, Acanceh, Coba, Tikal, Nakum.
SD2	17	21	27	1.235	Cempoala.
3/W	20	25	32	1.25	Viking. Xolalpan, Tepoztlan, Yucanudahui, Palenque, Tonina, Chacmultun, Labna(3), Macanxoc(2), Xelha+Cacakal(2), Nakum.
D	**3**	**4**	**5**	**1.333**	Teotihuacan Sun Py. Teotihuacan Moon Py. Xolalpan. Viking, Tula(3), Tenayuca(2), Tenochtitlan(2), Cempoala(3), Palenque(2), Piedras Negras, Agua Escondida(3), Chichen Itza, Chichen Itza, Uxmal(5), Uxmal(3*), Uxmal, Acanceh(2), Labna, Macanxoc(2), Nohoch Mul, Meco, Tikal.
G/2	13	19	23	1.461	Mitla.
L	10	15	18	1.500	Tetitla, Xochicalco, Tenayuca(2), Teopanzolco, Cempoala(2), Yaxchilan, Chichen Itza, Uxmal, Acanceh,
ΦA	10	16	19	1.600	Agua Escondida.
LVC	14	23	27	1.643	Macanxoc.
4/W	18	30	35	1.667	Tenochtitlan(2), Tajin(2), Chichen Itza, Chacmultun(2), Macanxoc(2).
GA/2	7	12	14	1.714	Cempoala, Macanxoc.
MA	4	7	8	1.75	Tzitzuntan, Palenque.
M	**8**	**15**	**17**	**1.875**	Cempoala(?),Palenque, S.Elena P.U., Chichen Itza, Uxmal(5), Labna, Kucikan.
2Q	4	8	9	2	Palenque, Piedras Negras, Chichen Itza, Chacmultun, Acanceh.
2Q	9	18	20	2	Tepantitla, Tula, Cempoala, Agua Escondida, Uxmal(4), Uxmal(2*), San Jose.
2Q	13	26	29	2	Tenayuca, Chichen Itza, Acanceh(4), Coba.
2QB	9	19	21	2.111	Cempoala, Palenque.
√5	9	20	22	2.222	Misantla, Yaxchilan, Uxmal, Copan.
WA	10	23	25	2.3	Acanceh, Copan.
W	**5**	**12**	**13**	**2.4**	Tenochtitlan(2), Yucanudahui, Misantla(3), Yaxchilan(2), Chacmultun, Copan.
WB	10	25	27	2.5	Yucanudahui, Cempoala, Tonina, Chacmultun, Acanceh, Meco, Zacpeten.
Φ²	5	13	14	2.6	Yaxchilan(2), Chichen Itza, Uxmal, Nohoch Mul.
2D	6	16	17	2.667	Agua Escondida, Chichen Itza, Uxmal, Chacmultun.
3Q	6	18	19	3	Meco.
GA	**7**	**24**	**25**	**3.429**	Chacmultun, Coba.

Conclusions

A total of 154 plans of Mesoamerican constructions were analysed and the results can be summarized as follows.

Presence of Squaring Triads and CQCs

Triads are present in all regions but, in the absence of evidence of combinations in harmonic schemes, no special role in the design can be attributed to them. Most likely they only served as squaring tools at the building yard. Nonetheless they appear to have been widely used, together with a few (but nonetheless significant) CQCs (Cuicuilco, Cempoala and possibly the Caracol of Chichen Itza). Table 2 contains a résumé.

Numbers

The monuments appear to be rich in calendrical integers, with percentages far beyond the expected 27%. The cultural significance (both cosmological and sociological) of the basic calendrical integers 9, 10, 13 and 20 has already been amply demonstrated, so that city-plans and layouts can be seen as sacred depositories of cosmological space-time connections (Lopez 1984; Sugiyama 1993; Aveni and Hartung 2000; Sprajc 2005 and references therein). This view is strongly confirmed by the numerical relationships found at Teotihuacán and Monte Albán between the spatial distances at the main monuments and the Tzolkin and Haab calendrical numbers.

The sheer number of calendrical integers found makes one wonder whether it was the dimensions of the architecture that were governed by the calendar or whether the numbers governed both calendars and architecture. If the calendars came first, then an undisputed explanation for the 13 x 20 and 18 x 20 structures has not yet been reached, although several reasonable but different putative explanations have been put forward (see e.g. Aveni 1989) If the numbers came first, the reason may possibly have rested on some special peculiarity of the numbers themselves. The only noteworthy numerical circumstance that I could figure out is that $18^2 + 26^2 = 1000$ ($18^2 + 26^2 = 10^3$).

A Geometrical Analysis of Mesoamerican Pre-Hispanic Architecture: Squaring Triads, Numbers, Length Units and the Calendar

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Units of length

Three length-units were found: a "Teotihuacán unit" **t** = 0.575 ±0.002 m; a "Monte Albán unit" **ma** = 0.504 ±0.007 m; and a "Chichen Itza unit" **ci** = 0.665 ±0.015 m. Their geographical distribution is illustrated in Plate VI: Fig. 11.

Other values for length-units have previously been hypothesized. As reviewed in Sugiyama (1993), all are outside the cubit-like range of 40-70 cm. Correspondences with **t** can be found through calendrical multipliers or ratios: the Drewitt value of 57 m is 100 x **t** (to within 1%); his 322 m large unit equals 560(=20x28)**t** while (28/20)**t** equals the 80.5 cm value of Drucker and Drewitt. Séjourné proposed a value of 60 cm, which is close to 104(=13x8)**t** to within 0.3%. Sugiyama's proposed value of 82.3 cm equals (13/9)**t** within 1%. O'Brien and Christiansen's 147-cm unit for the Maya Pooc-style sites equals (20/9)**ci** (1%).

The analysis is in no way exhaustive. The results are intended to be "virtual" and wait to be confirmed (or rejected) on the basis of more accurate measurements. What I have presented is a "progress report": much is left to be done and much more to be understood. Nonetheless, I believe that the calendrical-numbers-to distances connection found at Teotihuacán and Monte Albán is unlikely to be disproved and that, in the years to come, this connection will very possibly indicate the road to take in order to arrive at a coherent comprehension of the length-units, numerical repertories and architectural geometries used by the ancient Mesoamerican builders.

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GEOMETRINĖ MEZOAMERIKOS IKIKOLONIJINĖS ARCHITEKTŪROS ANALIZĖ: PITAGORO SKAIČIŲ TREJETAI, ILGIO VIENETAI IR KALENDORIUS

Marcello Ranieri

Santrauka

Straipsnis tęsia Mezoamerikos senovinės geometrijos tyrinėjimus, kurie paremti senųjų civilizacijų architektūros analize. Analizuojant duomenis – erdvinius archeologinių brėžinių komponentus – naudota CAD (Computer Aided Design) programinė įranga. Tyrimas pateikė kiekybinę informaciją apie formas, panaudotus skaičius ir ilgio matus.

Tyrimo metu buvo konstatuoti trijų rūšių ilgio matai: Teotihuacán (t ≈ 0,58 m), Monte Albán (ma ≈ 0,50 m) ir Chichen Itza (ci ≈ 0,66 m) ilgio vienetai. Vienetai t ir ma yra akivaizdžiai susiję su atstumais tarp svarbiausių paminklų, o šie savo ruožtu – su Tzolkin kalendoriniais skaičiais 13–20 Teotiuakane ir Haab kalendoriniais skaičiais 18–20 Monte Albanoje.